Gapless to gapless phase transitions in quantum spin chains

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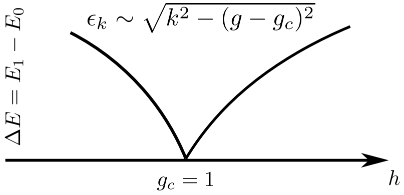
Introduction

Correlation length ξ is related to gap:

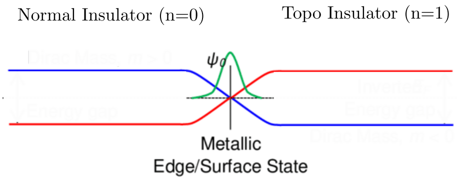
$$m_G \sim \xi^{-1}$$

Continuous phase transition $\Longleftrightarrow$ Closing - opening of energy gaps

e.g. Transverse field Ising:



e.g. Bulk-edge Correspondence



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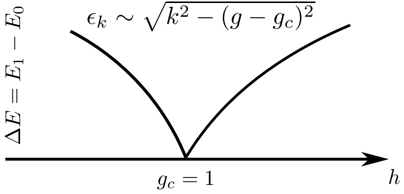

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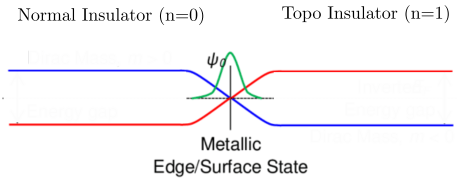
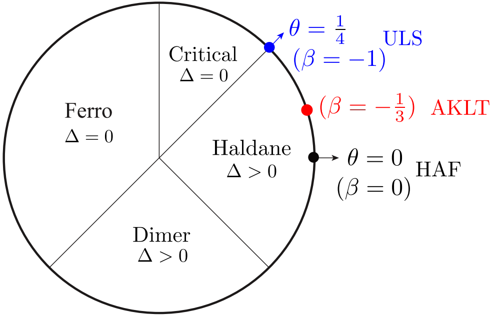


gapless to gapless transition in spin-1 chain under external field

1D Bilinear-Biquadratic (BLBQ) Model

$$H_{BLBQ} = \sum_{\langle ij \rangle} \cos\theta (\mathsf{S}_i \cdot \mathsf{S}_j) + \sin\theta\, (\mathsf{S}_i \cdot \mathsf{S}_j)^2$$



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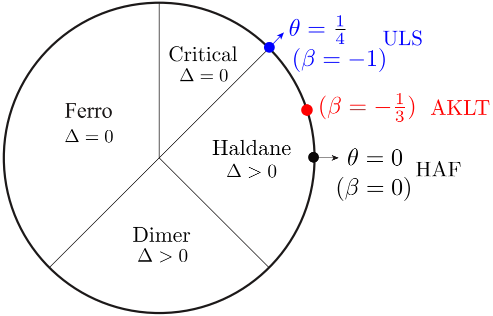

With magnetic field:

$$H = \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + \beta\,(\mathbf{S}_i \cdot \mathbf{S}_j)^2 + \boxed{h \sum_i S_i^z}$$

Note external field is a good Quantum number

1. *AKLT:*
   SPT $\rightarrow$ Partially Polarized
   $\rightarrow$ Polarized

2. *ULS:*
   No protection of gap
   Partially Polarized $\longrightarrow$ Polarized **?**

Phase Diagram

ULS: *Gapless* $\rightarrow$ Gapless $\rightarrow$ Polarized

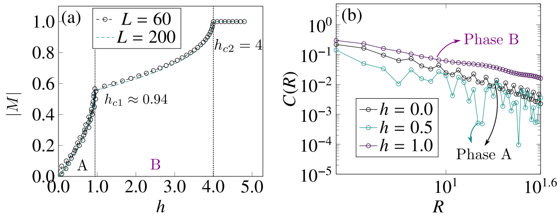

Figure: (a) Magnetization vs field. (b) Real space correlation function in 2 phases

Phase Diagram

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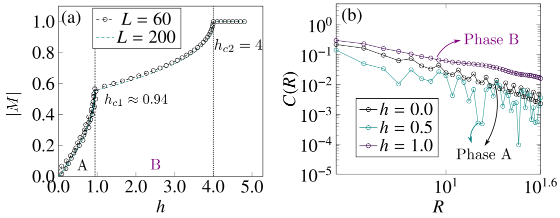

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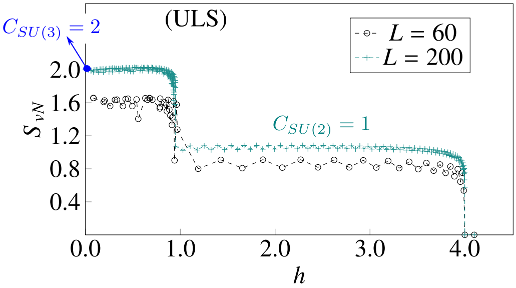
von-Neumann Entropy of ULS



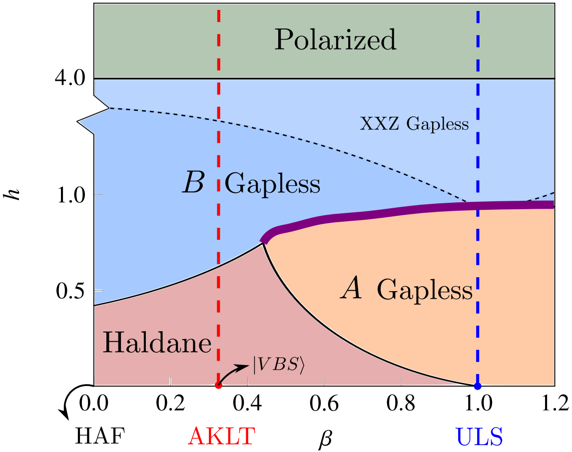


Figure: Schematic phase diagram of BLBQ with field, obtained from DMRG under OBC based on Littlewood et al (1999)

How to distinguish Phase A and B?

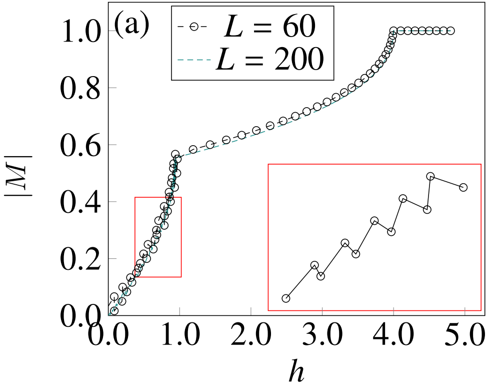


Figure: Magnetization of ULS under field

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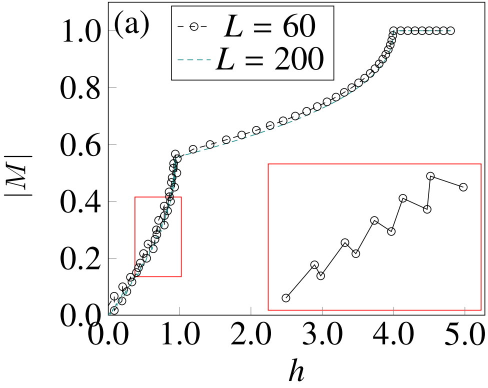


Figure: Magnetization of ULS under field

Fermionic spinon of spin-1:

$$[\hat{N}_m, H] \equiv \left[\sum_i b^\dagger_{i,m} b_{i,m}, H \right] = 0$$

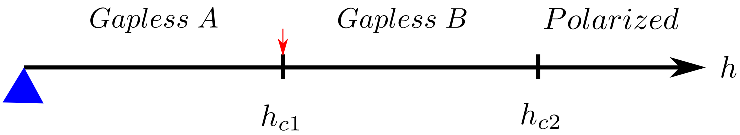
For $h > 0$, SU(3) $\rightarrow$ U(1) $\times$ U(1).
3 conserved charges are intact in $U(1) \times U(1)$.

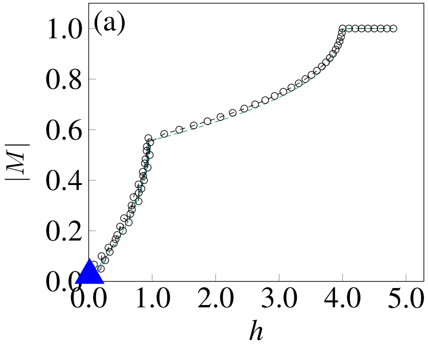
2 independent competing charges:

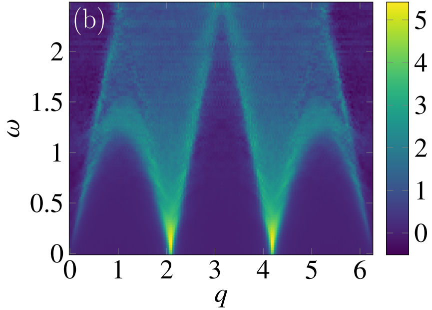
$$E = E_{ULS}(N_1, S_z) + hS_z$$

Transition to phase B as N_1 is exhausted.

Dynamical Structure Factors







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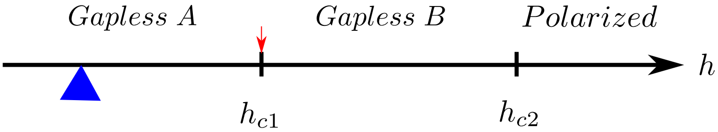

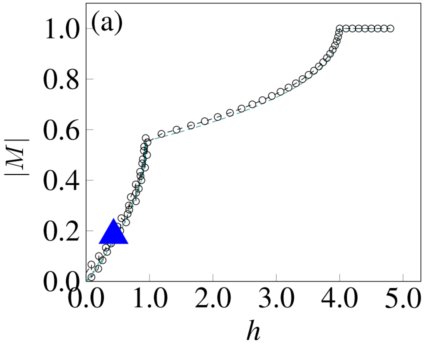

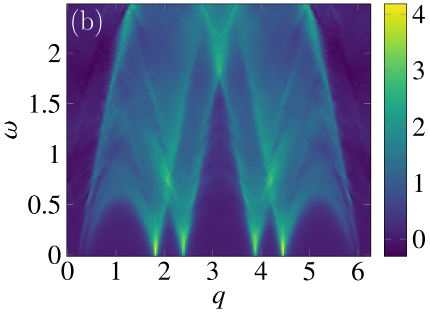

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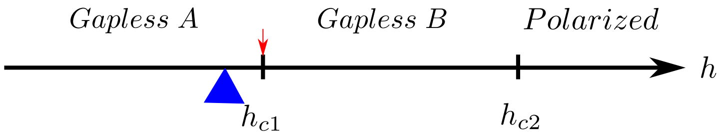

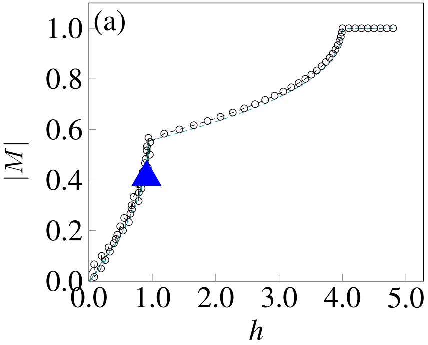

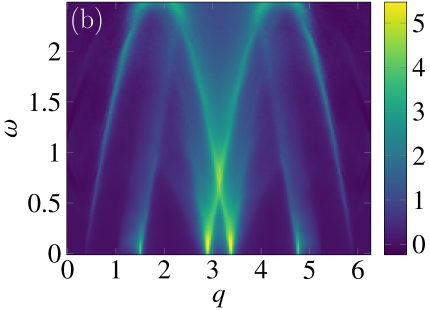

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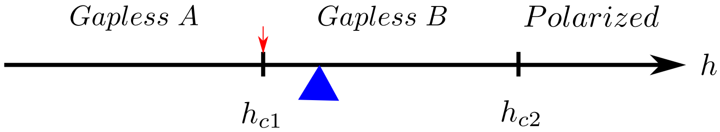
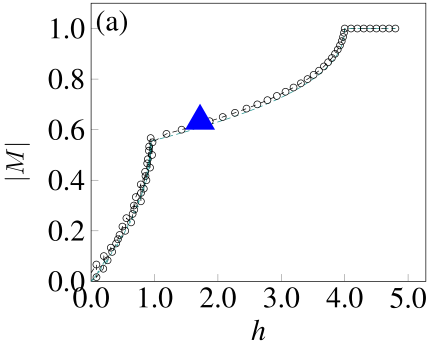
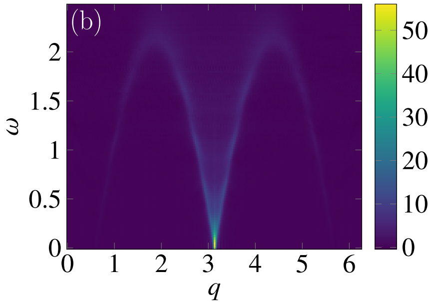
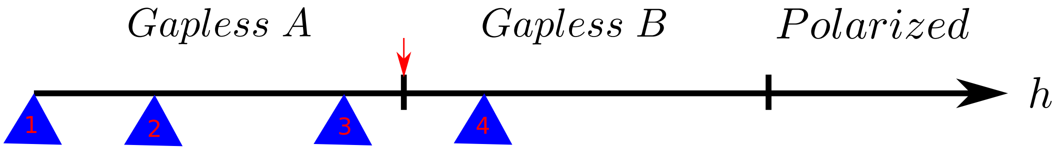
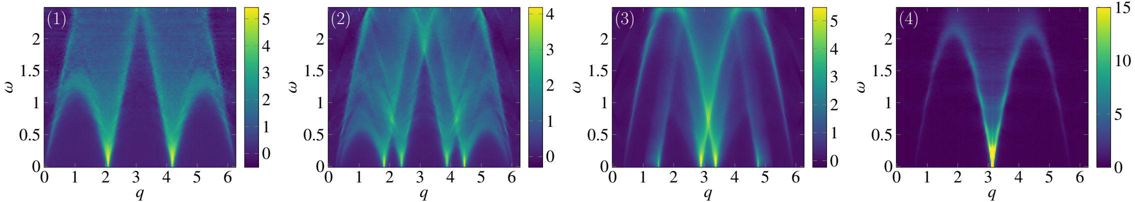










transfer of SU(3) chiral fermions* and de-population of spinon bands

Candidate: $5d^4$ transition-metal Mott insulators (PRB 101, 155112 (2020))

*Affleck, Nuclear Physics B 265, 409-447 (1986). *ibid* 305, 582-596(1988)

Summary

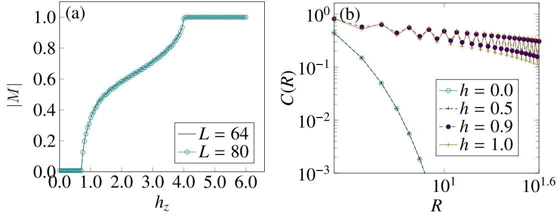
1. A gapless to gapless transition in ULS under a field

$$H = \sum_{\langle ij \rangle} \mathrm{S}_i \cdot \mathrm{S}_j + (\mathrm{S}_i \cdot \mathrm{S}_j)^2 + h \sum_i S_i^z$$

2. Statics: Jump in central charges
3. Dynamics: Bifurcation of gapless (quasi) modes $\iff$ Interplay of SU(3) chiral fermions and de-population of spinon bands
4. Possibly shows up in $5d^4$ transitional metal oxides.

Phase Diagram

AKLT: Gapped → Gapless → Polarized



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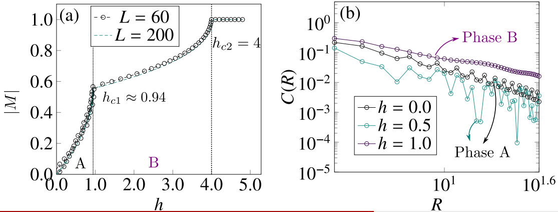
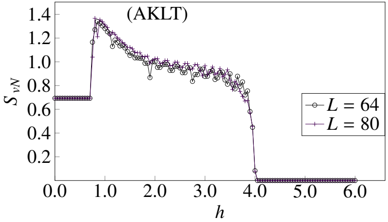


von-Neumann Entropy of AKLT



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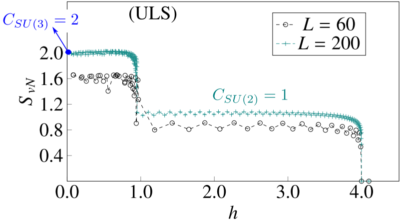

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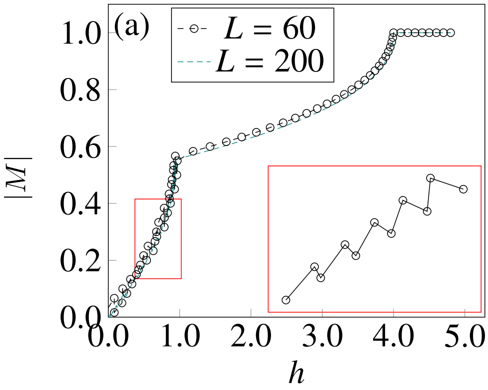


Figure: Magnetization of ULS under field

SU(3) Symmetry at $h = 0$:

$$\mathsf{S}_i \equiv \psi_i^\dagger \mathsf{S}_i \psi_i, \quad \psi_i = (b_{i,1}, b_{i,0}, b_{i,-1})$$

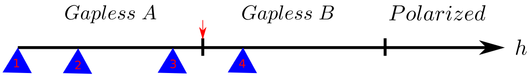
Explicit SU(3):

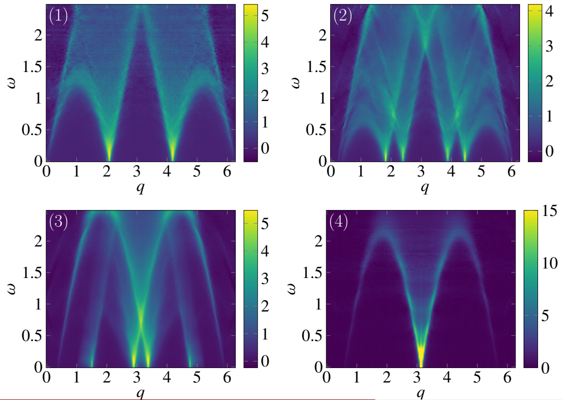
$$H_{ULS} = -\sum_{\langle ij\rangle;mm'} b^\dagger_{i,m} b_{j,m} b^\dagger_{j,m'} b_{i,m'}$$

3 conserved charges in ULS ($m = 1, 0, -1$):

$$[\hat{N}_m, H_{ULS}] \equiv \left[\sum_i b^\dagger_{i,m} b_{i,m}, H_{ULS}\right] = 0$$

Interplay between SU(3) chiral fermions*





$$d_{n,\alpha} \approx f_{L,\alpha}(x)e^{-ik_\alpha x} + f_{R,\alpha}(x)e^{ik_\alpha x}$$

k_α: fermi momenta of spinon-α

$$S^+ \approx f^\dagger_{R,1} f_{L,0} e^{-i(k_1+k_0)x} + f^\dagger_{R,0} f_{L,-1} e^{-i(k_0+k_{-1})x}$$

$N_1 \to 0 \quad \downarrow \quad k_1 \to 0$

$$S^+ \approx f^\dagger_{R,0} f_{L,-1} e^{-i\pi x}$$

Transition by spinon band de-population

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