Anyon, fractionalization, and their detection

Shi Feng

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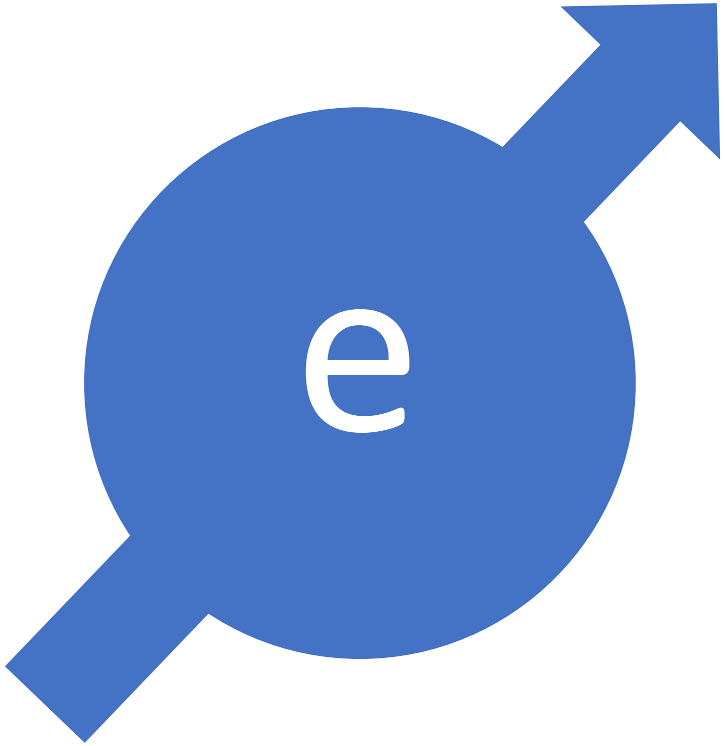
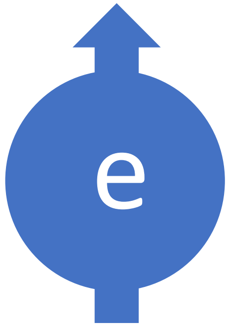
Feng, S., Agarwala, A., Bhattacharjee, S., & Trivedi, N. (2022).
Anyon dynamics in field-driven phases of the anisotropic Kitaev model.
arXiv.2206.12990

Qubit for Quantum Computers

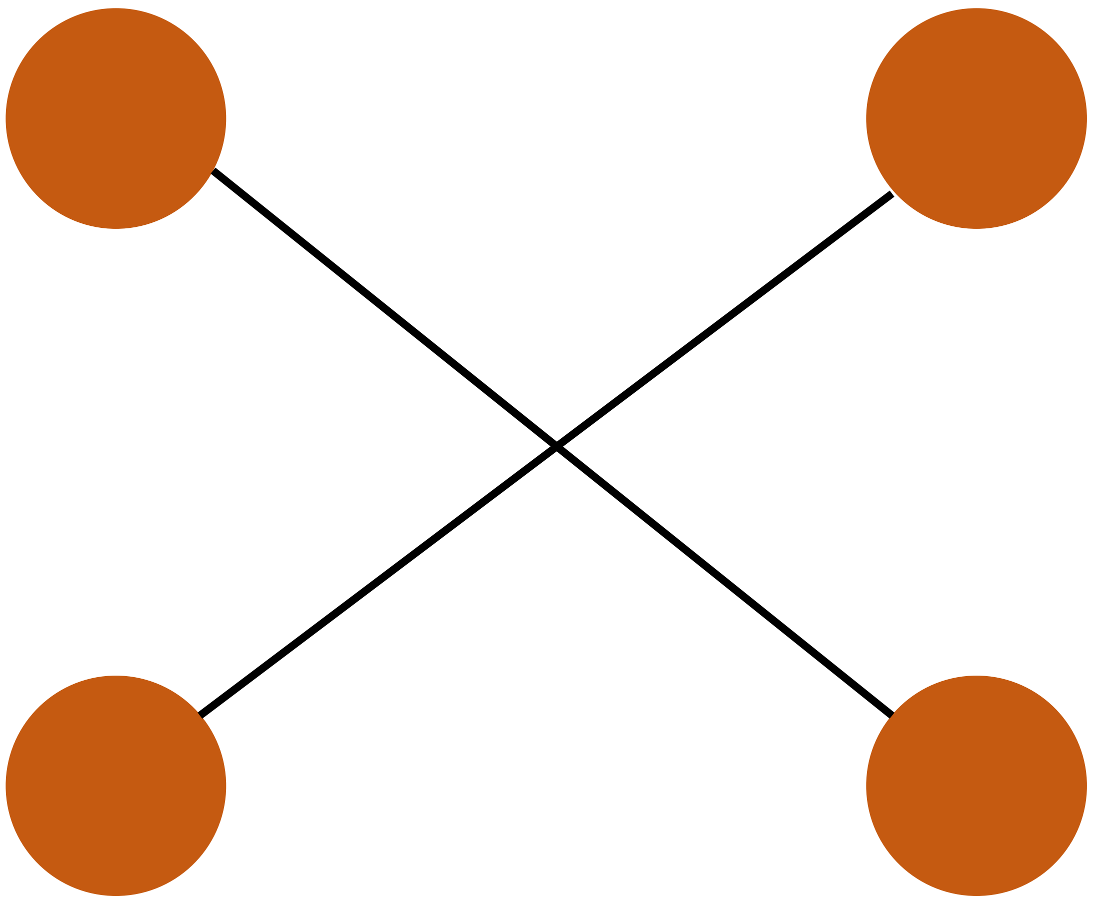


Spin as Quantum Bit (Qubit):



$$|\uparrow\rangle \sim |\,0\rangle$$

$$|\downarrow\rangle \sim |\,1\rangle$$



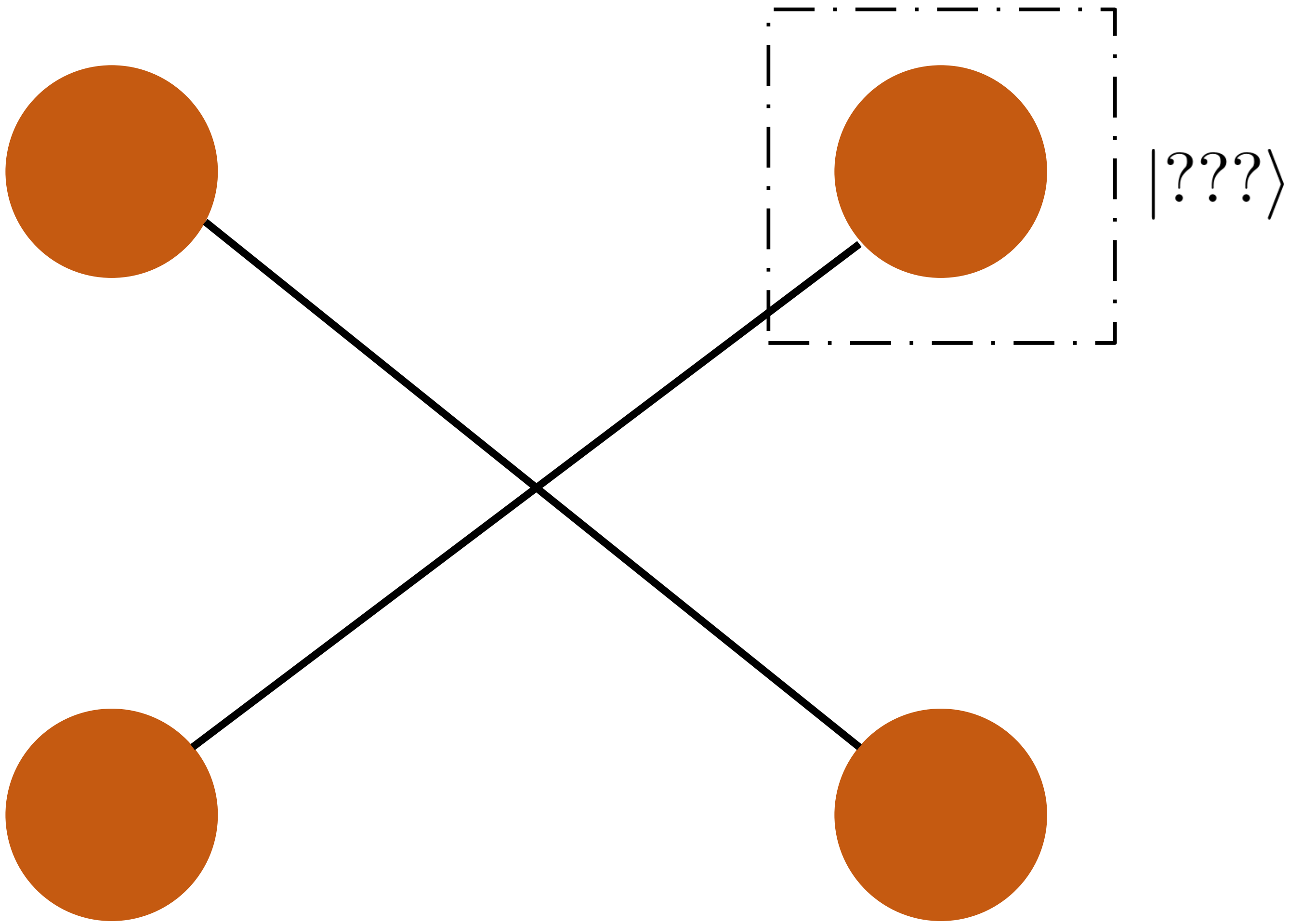
Fractionalization into Anyons

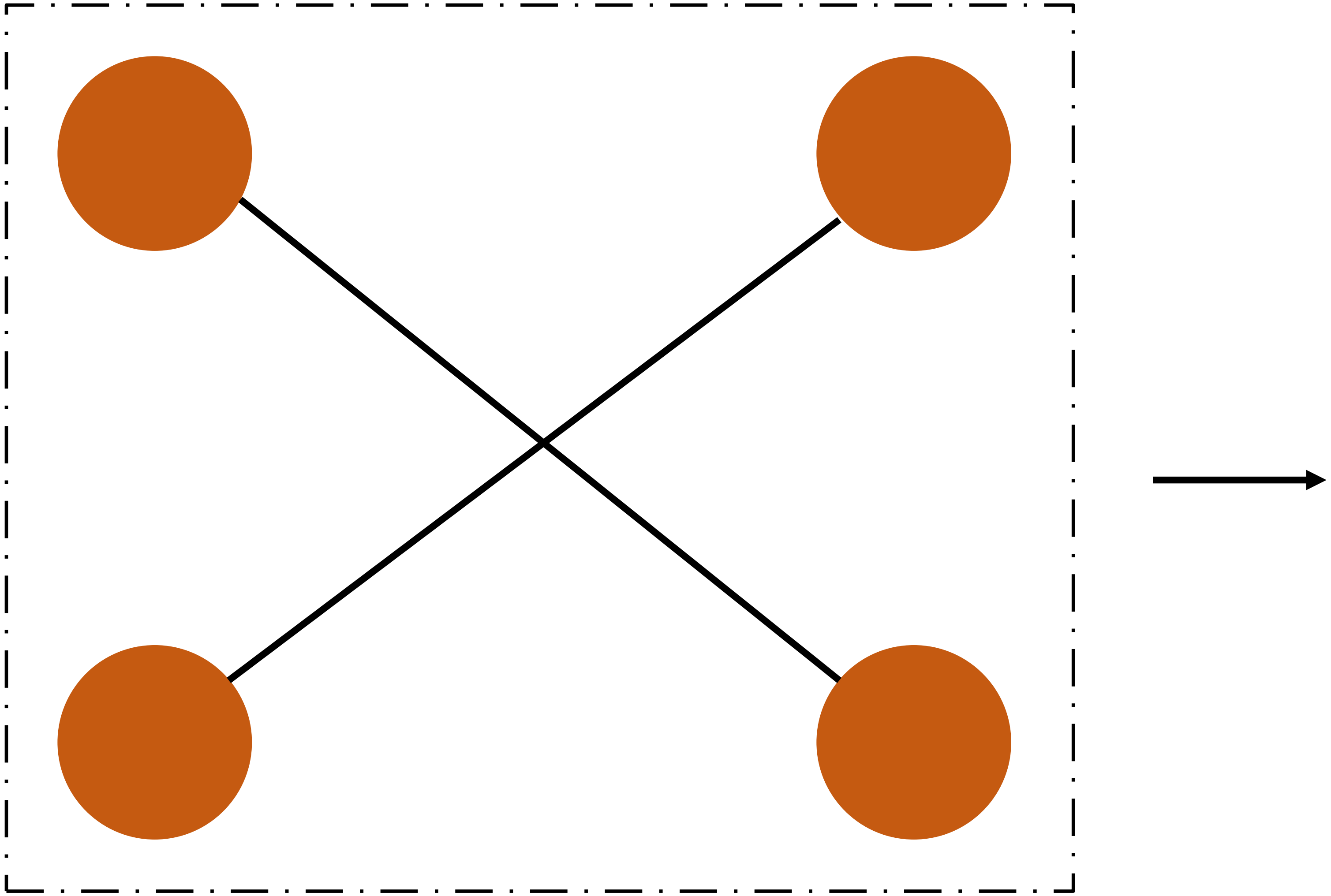
SU(2) Lie Algebra for spin-1/2:

$$[S^x, S^y] = iS^z$$

$$[S^y, S^z] = iS^x$$

$$[S^z, S^x] = iS^y$$


$|???\rangle$



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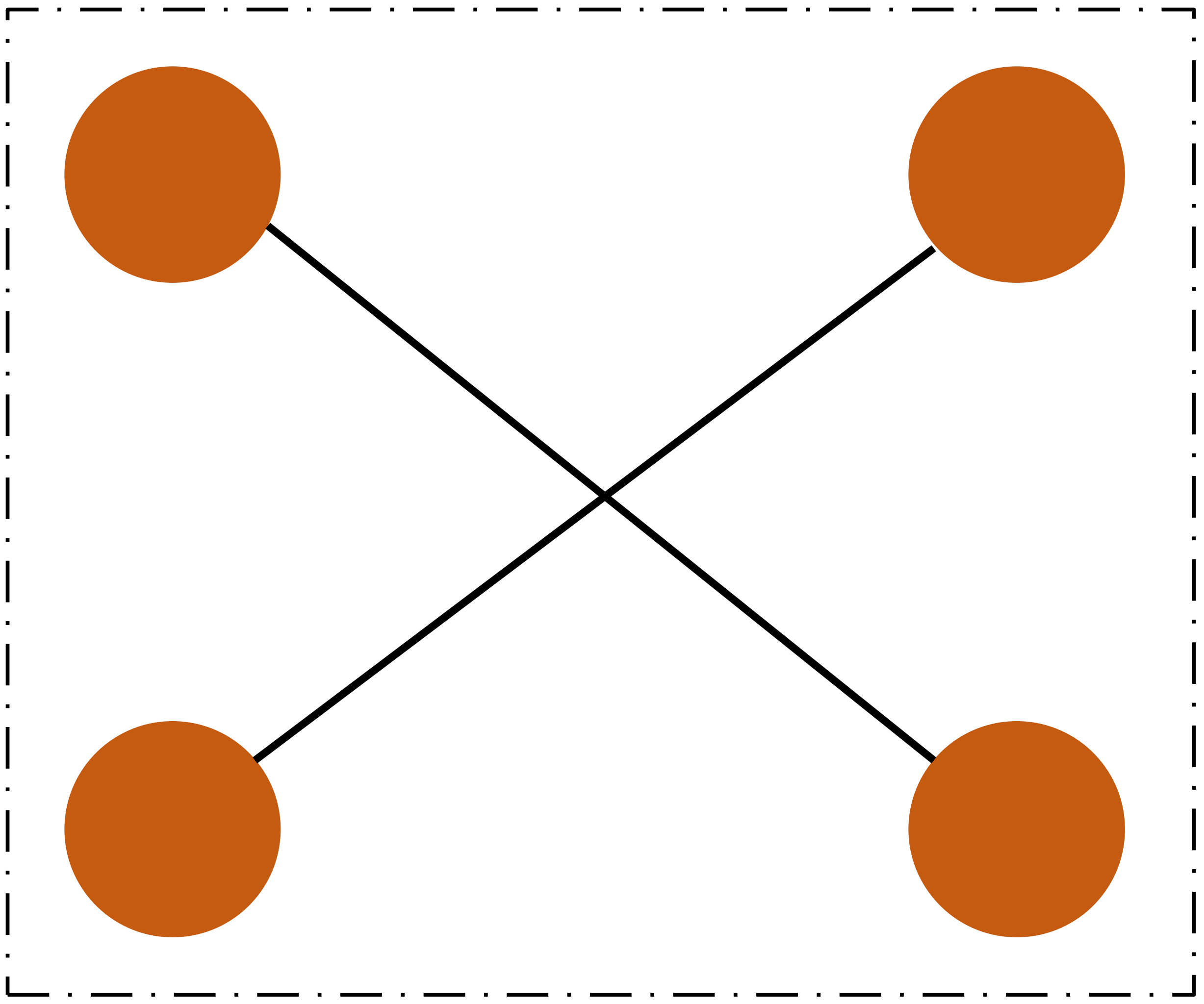

$$|\uparrow\rangle \sim |\,0\rangle$$

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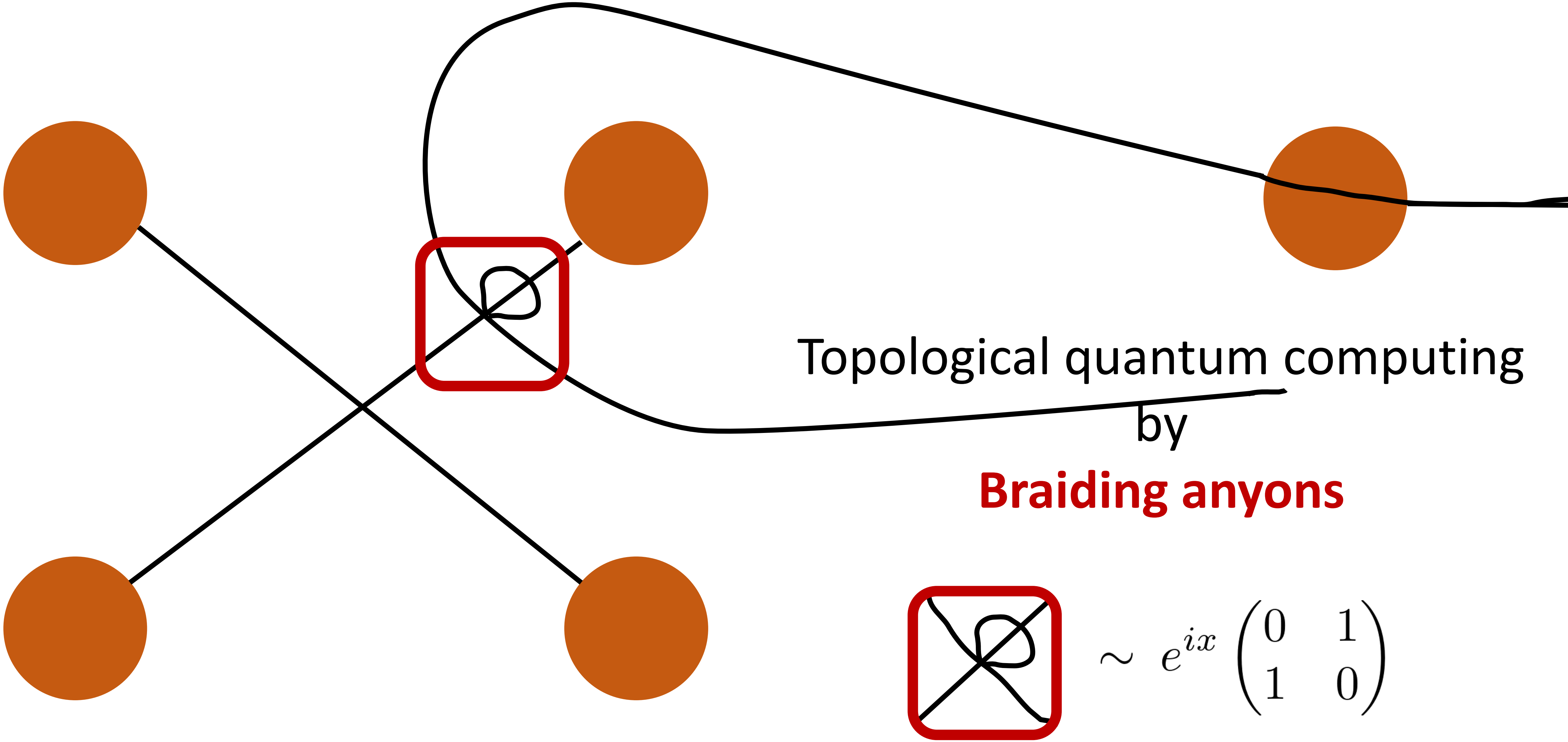
Immune to local perturbation



Topological quantum computing

by

Braiding anyons

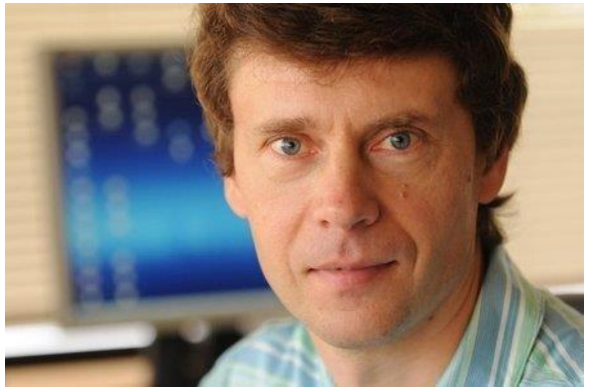
$\sim e^{ix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

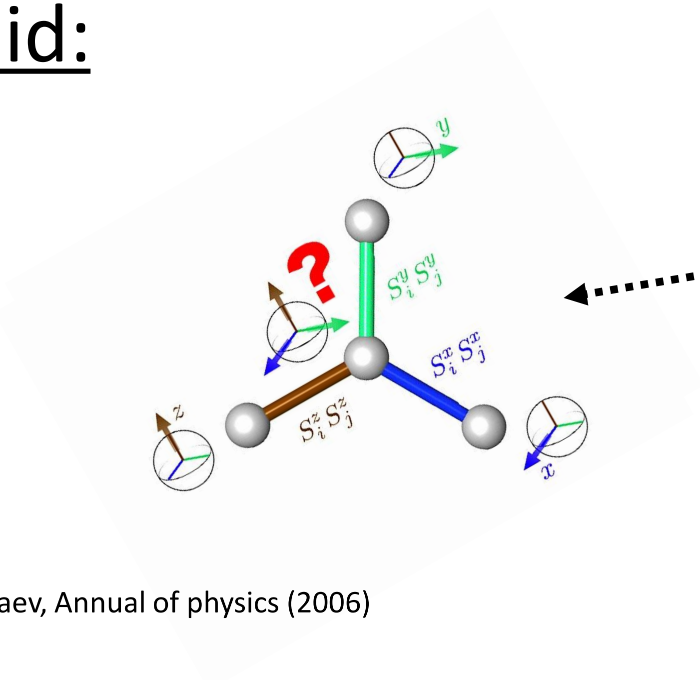
Where to find …

and

How to detect

Quantum Spin Liquid:



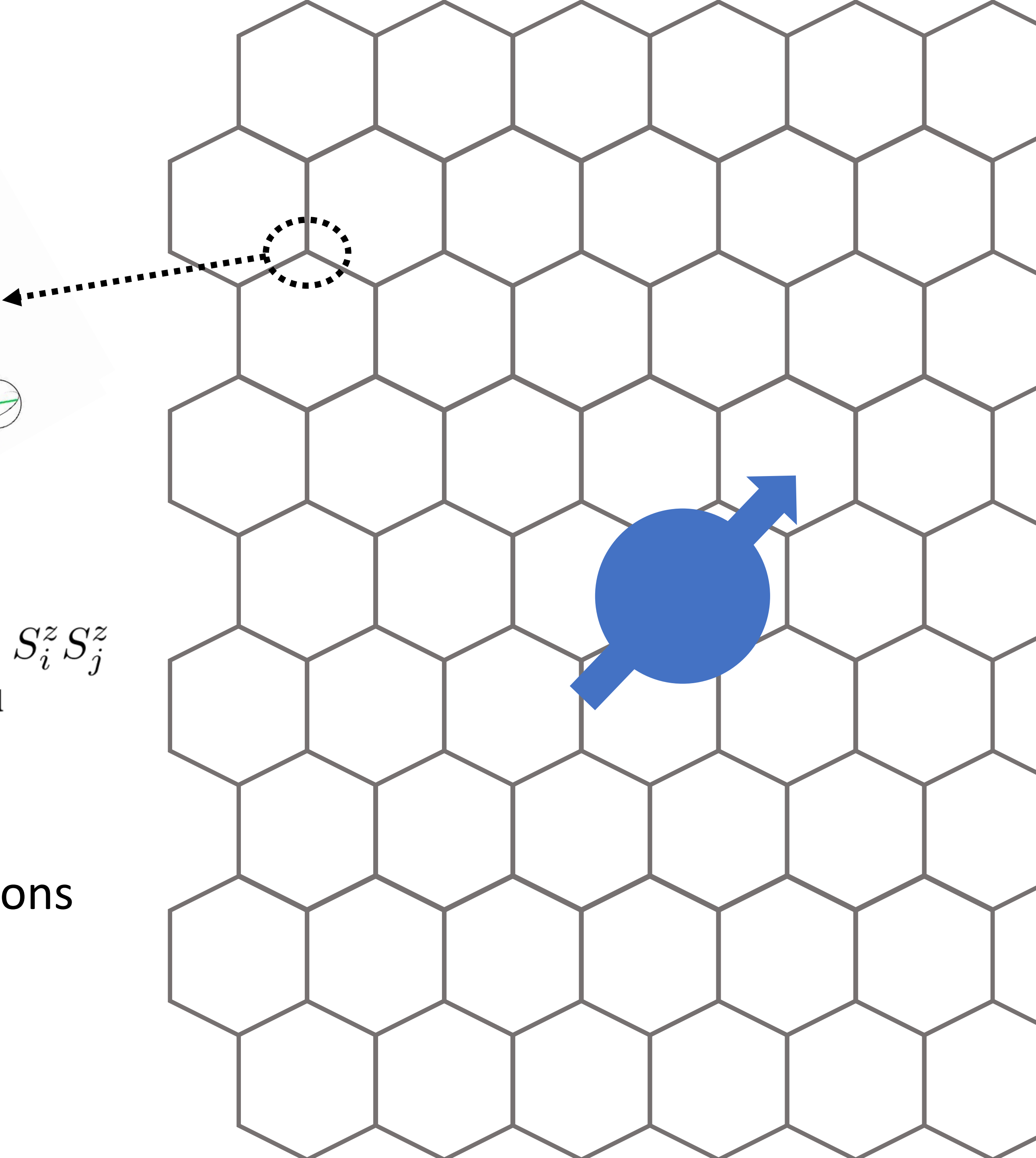


A. Kitaev, Annual of physics (2006)

$$H_K = K_x \sum_{\text{x bond}} S_i^x S_j^x + K_y \sum_{\text{y bond}} S_i^y S_j^y + K_z \sum_{\text{z bond}} S_i^z S_j^z$$

Fractionalization into Anyons:

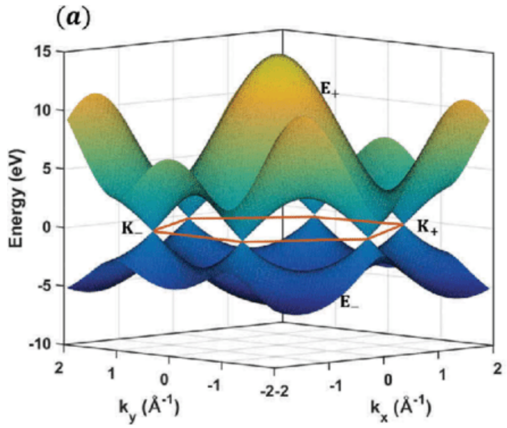
- ● Itinerant (Majorana) fermions
- ↻ Static Z2 flux or vortex

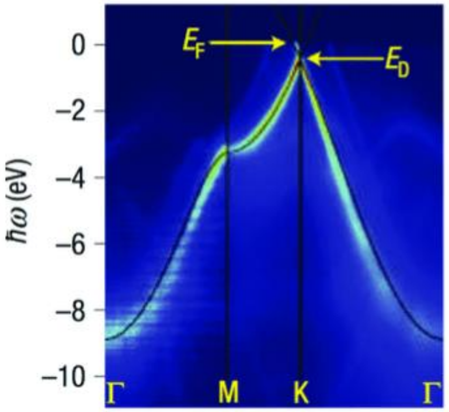


How to detect fractionalized particles?

Intact particles like electron or magnon:

$$H(k) \sim \sum_k \omega(k) c_k^\dagger c_k$$





Graphene band structure.
Energy spectrum measured by ARPES

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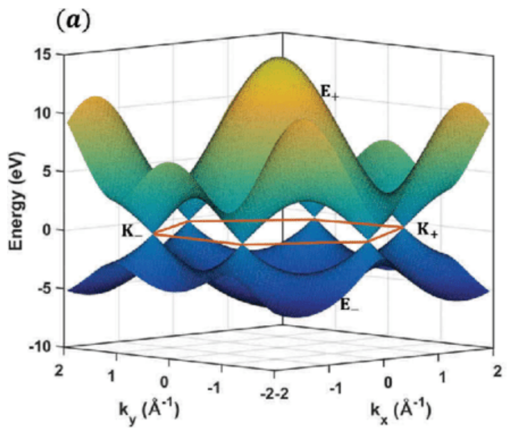

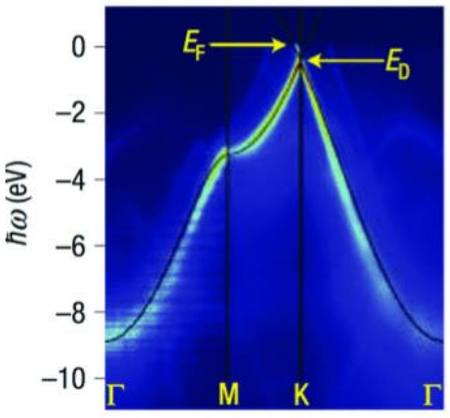

Graphene band structure.
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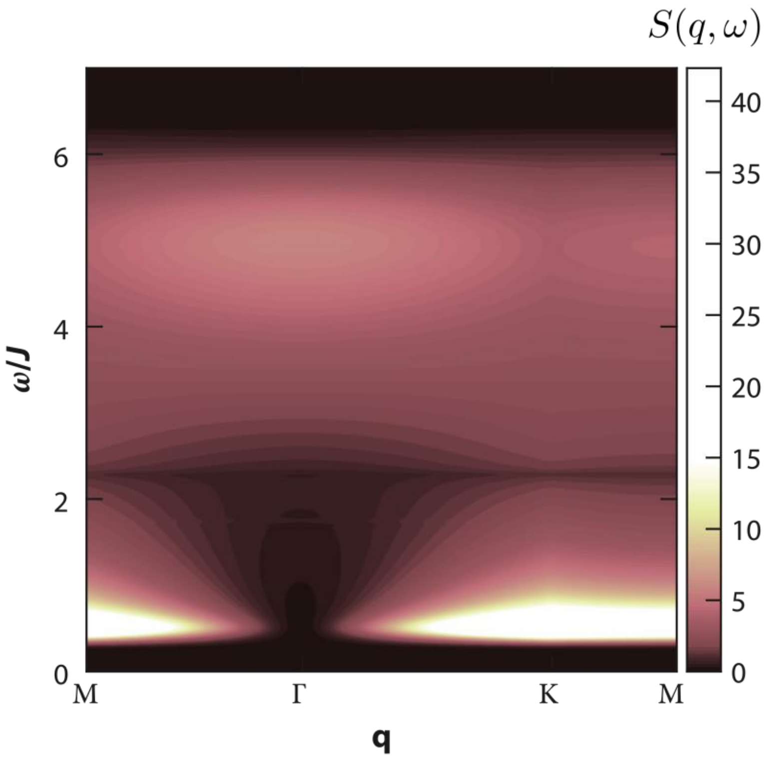
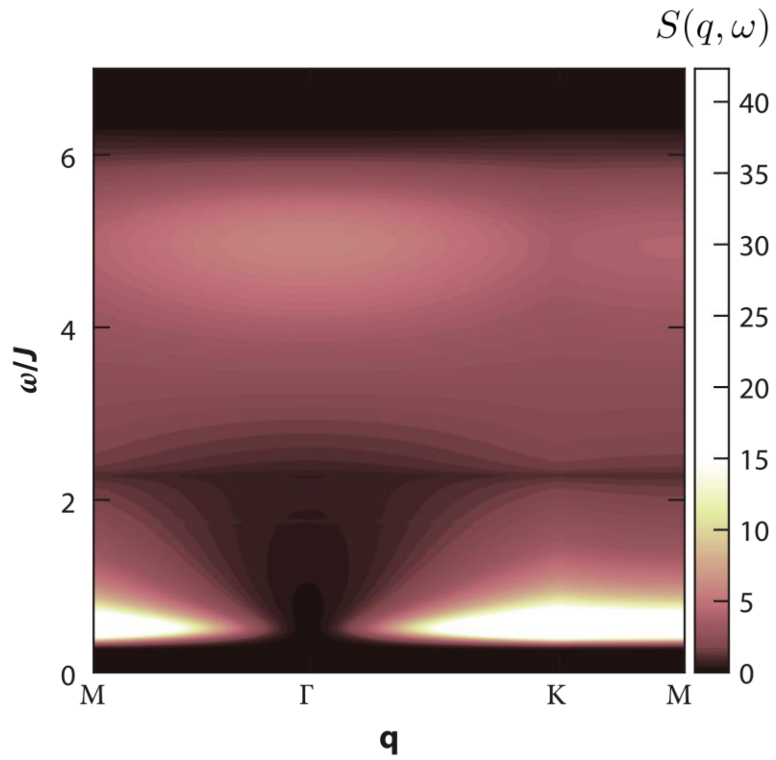
Spectrum by fractionalized particles



Dynamical spin structure factor of Kitaev model

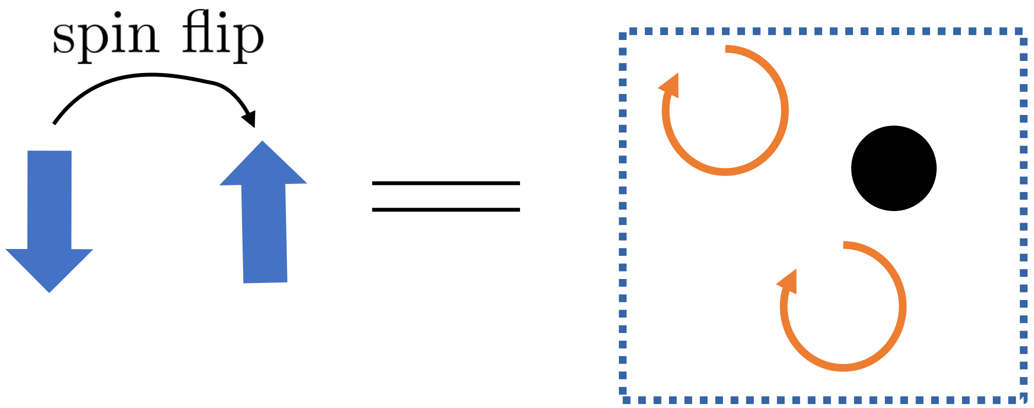
How to detect fractionalized particles?

Spectrum by fractionalized particles



Dynamical spin structure factor of
Kitaev model (in momentum and energy)

Spin excitation dissolves into an amalgam of different fractionalized particles



Broad Fuzzy signatures
Hard to distinguish from Disorder or Thermal noise.

Sharp, definitive signature of anyons?

Low-energy anyons at large Kz limit

$$H = K \sum_{\text{x bond}} S_i^x S_j^x + K \sum_{\text{y bond}} S_i^y S_j^y + K_z \sum_{\text{z bond}} S_i^z S_j^z$$

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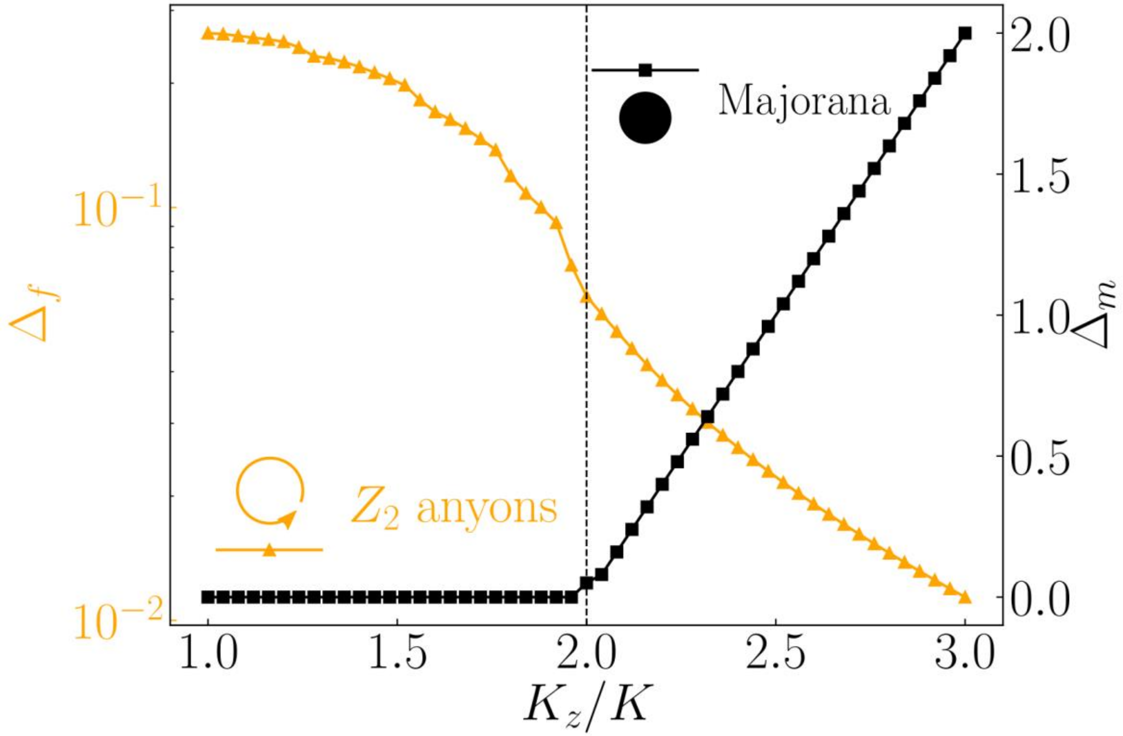

Separation of energy scales

Low-energy anyons at large Kz limit

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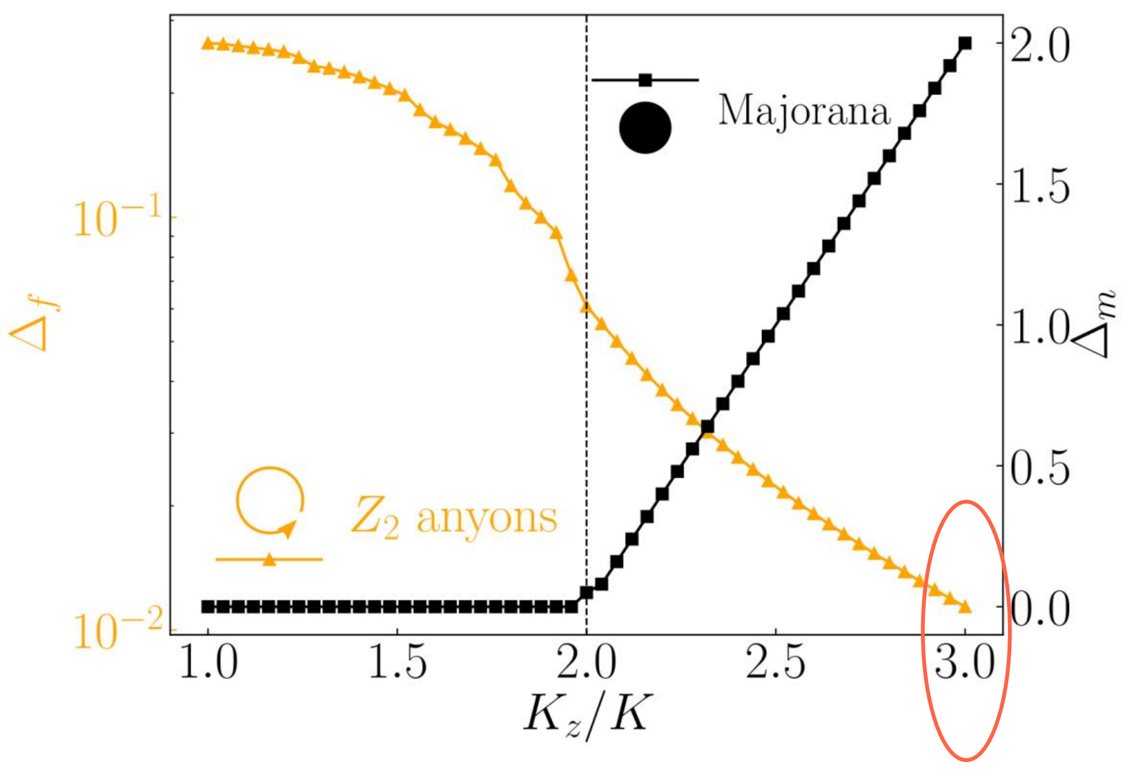

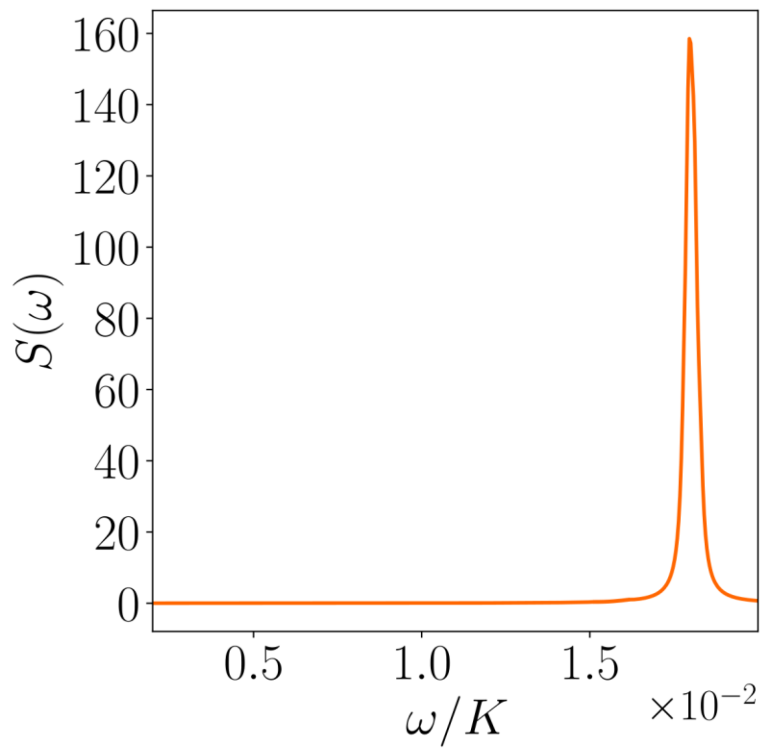

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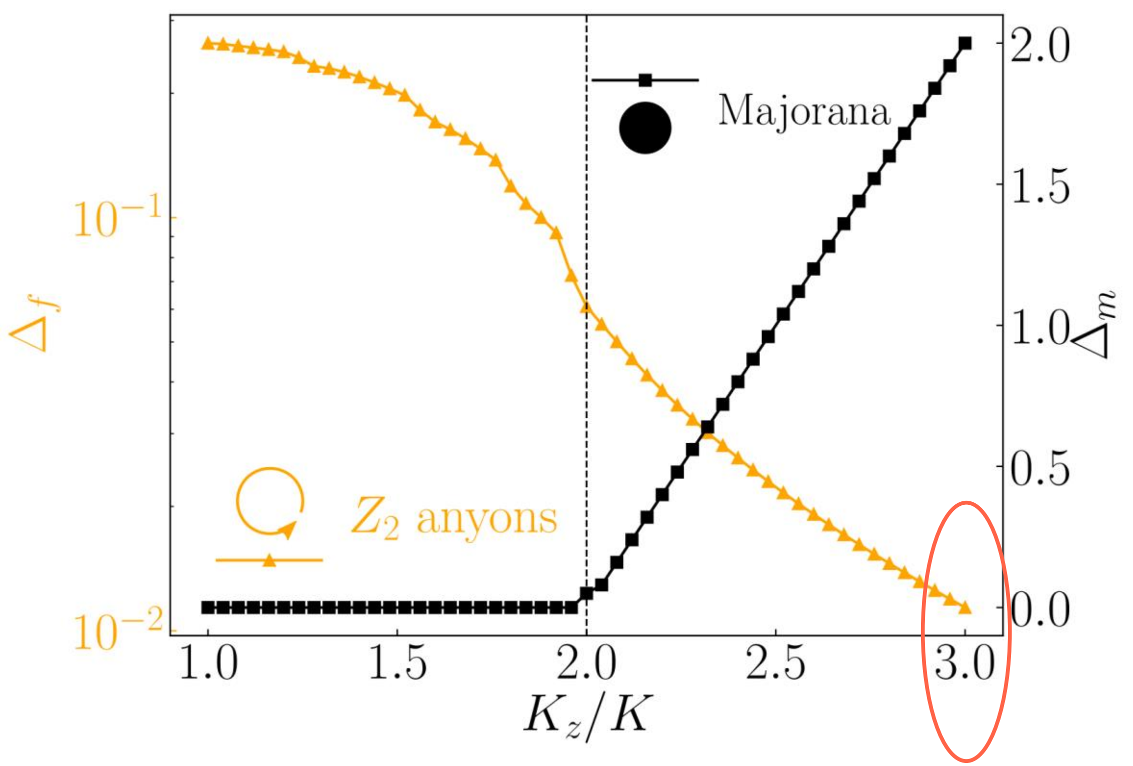

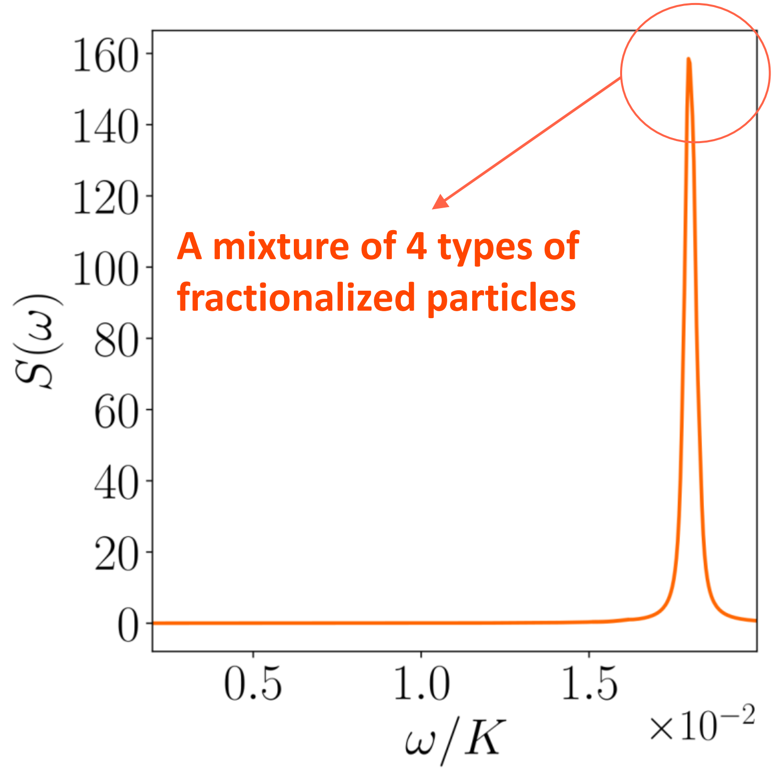

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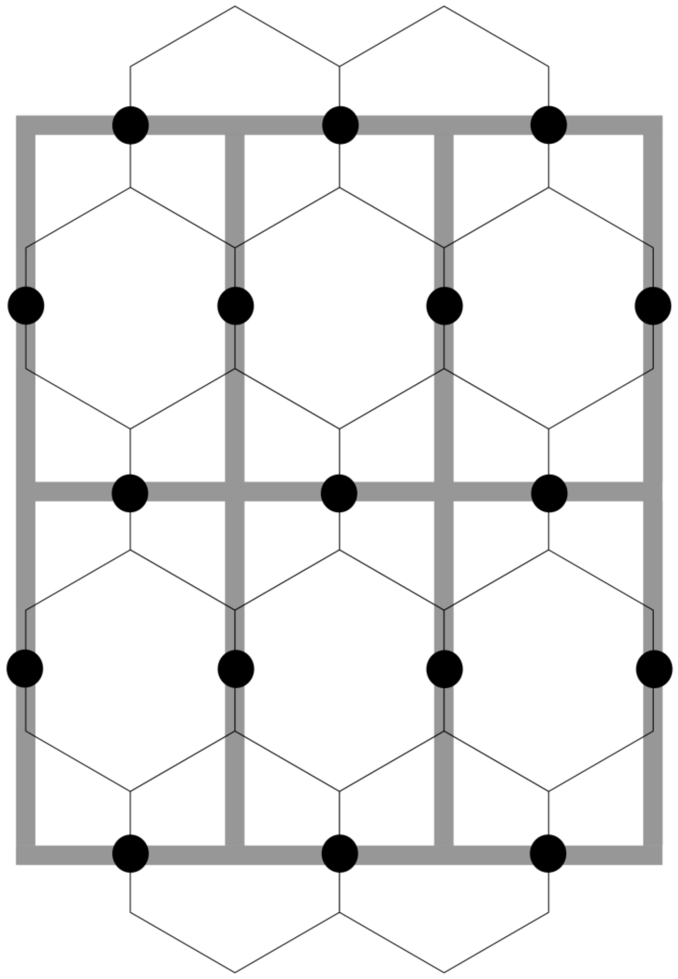

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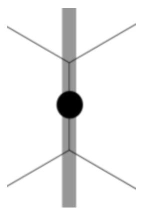
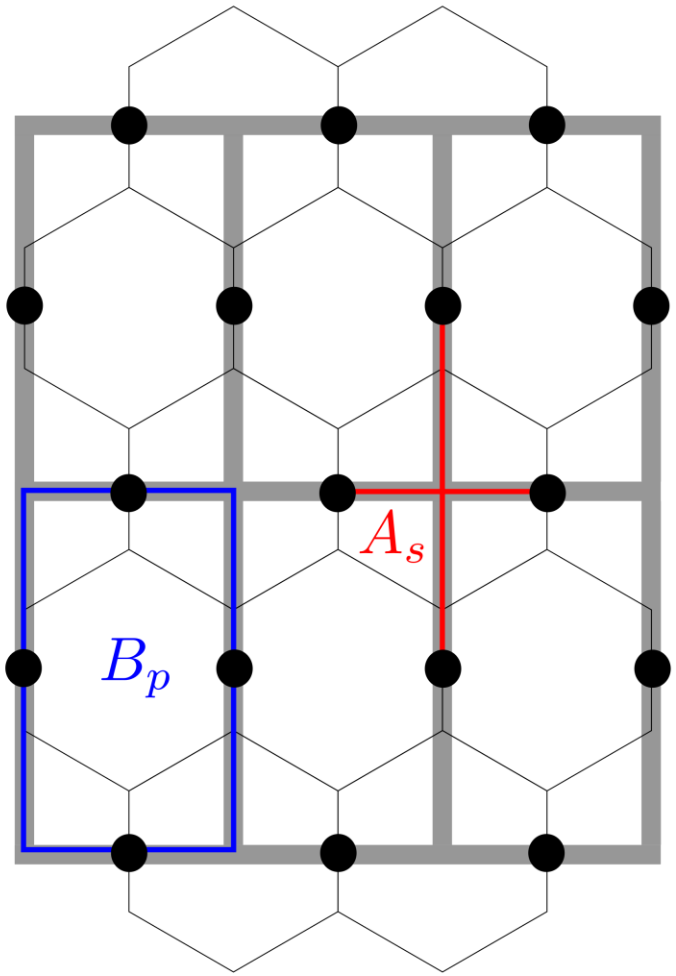


$$= \; |\uparrow\downarrow\rangle \text{ or } |\downarrow\uparrow\rangle \longrightarrow \tau$$

Low-energy anyons at large Kz limit



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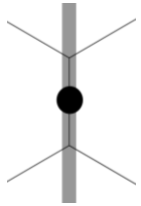
 $= \quad |\uparrow\downarrow\rangle$ or $|\downarrow\uparrow\rangle \longrightarrow \tau$

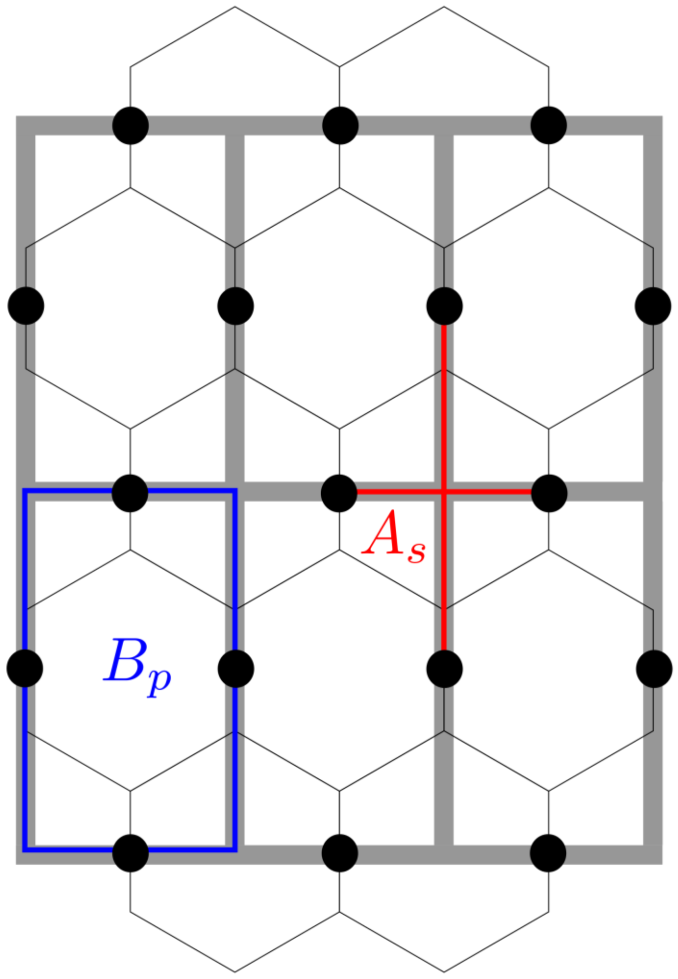
Fourth order perturbation theory:

$$H_{\text{eff}} \sim -\sum_s A_s - \sum_p B_p$$

$$A_s = \prod_{i \in s} \tau_i^x \qquad B_p = \prod_{i \in p} \tau_i^z$$

A. Kitaev (2006)

Low-energy anyons at large Kz limit



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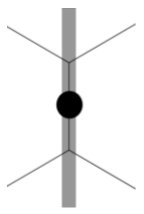

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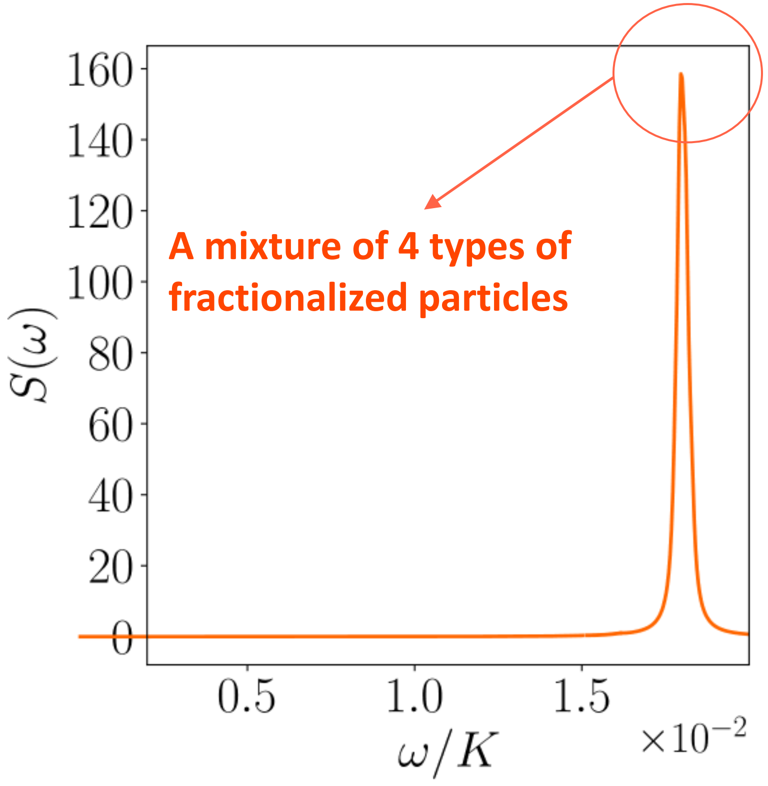
4 types of non-trivial anyons:

- **Gauge electric charge (As)**
- **Gauge magnetic charge (Bp)**
- **Composite anyon (As x Bp)**
- **Majorana fermions**

Low-energy anyons at large Kz limit



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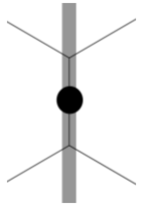

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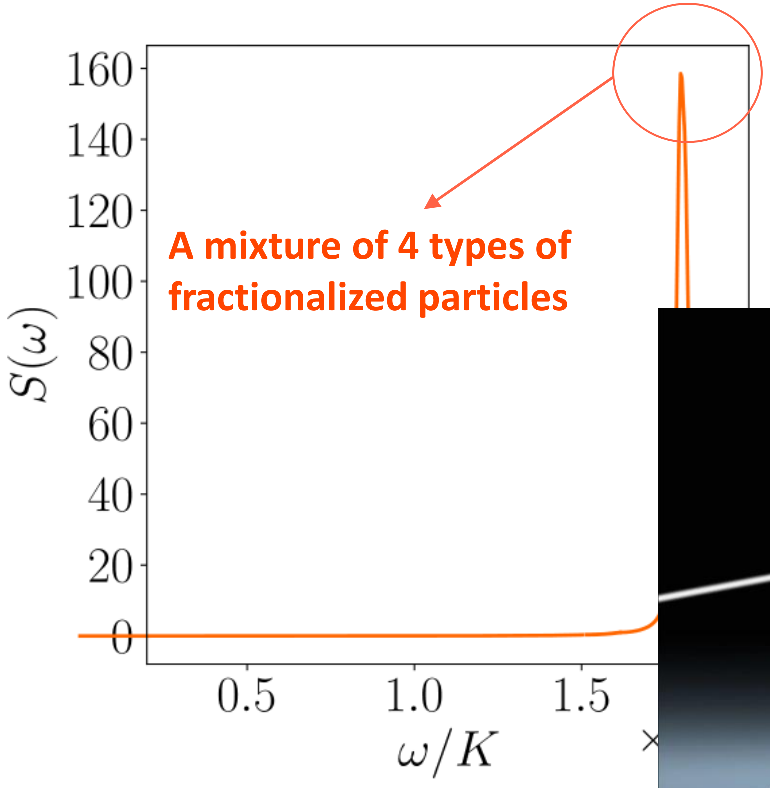
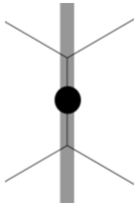
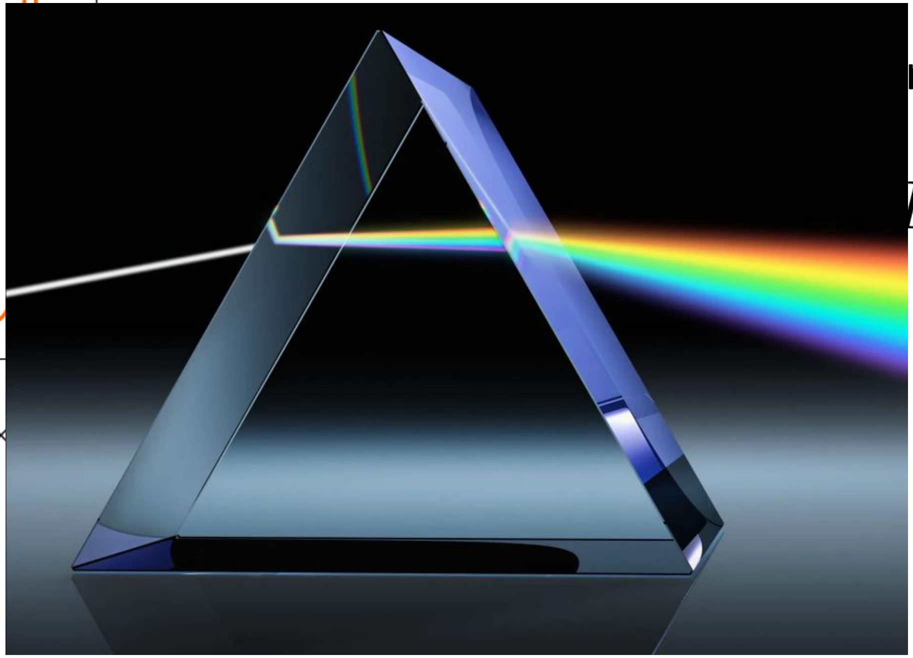


 $= |\uparrow\downarrow\rangle$ or $|\downarrow\uparrow\rangle \longrightarrow \tau$



ry:

B_p

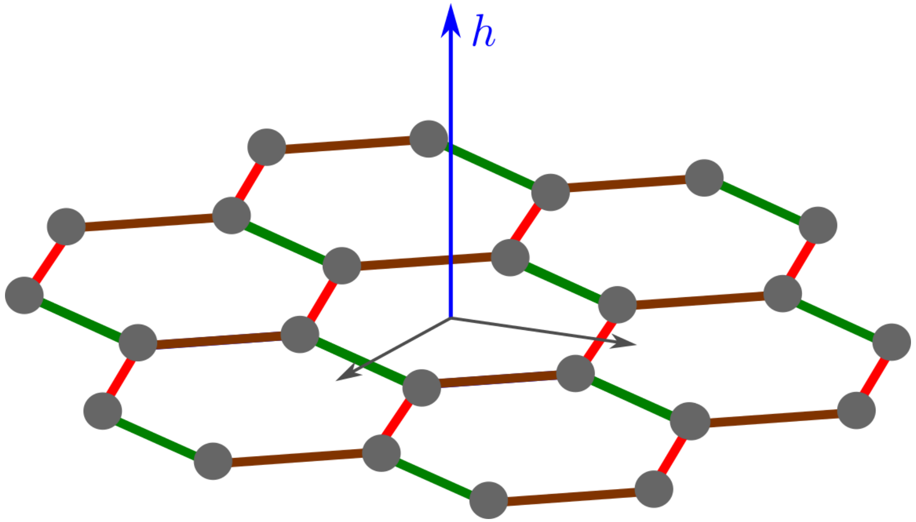
τ_i^z

4 types of non-trivial anyons:

- **Gauge electric charge (As)**
- **Gauge magnetic charge (Bp)**
- **Composite anyon (As x Bp)**
- **Majorana fermions**

Magnetic-field-induced anyon dynamics

$$H = K \sum_{\text{x bond}} S_i^x S_j^x + K \sum_{\text{y bond}} S_i^y S_j^y + K_z \sum_{\text{z bond}} S_i^z S_j^z - \vec{h} \cdot \sum_i \vec{S}_i$$

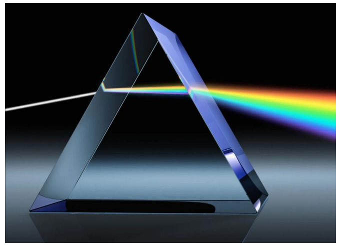


Second order perturbation theory:

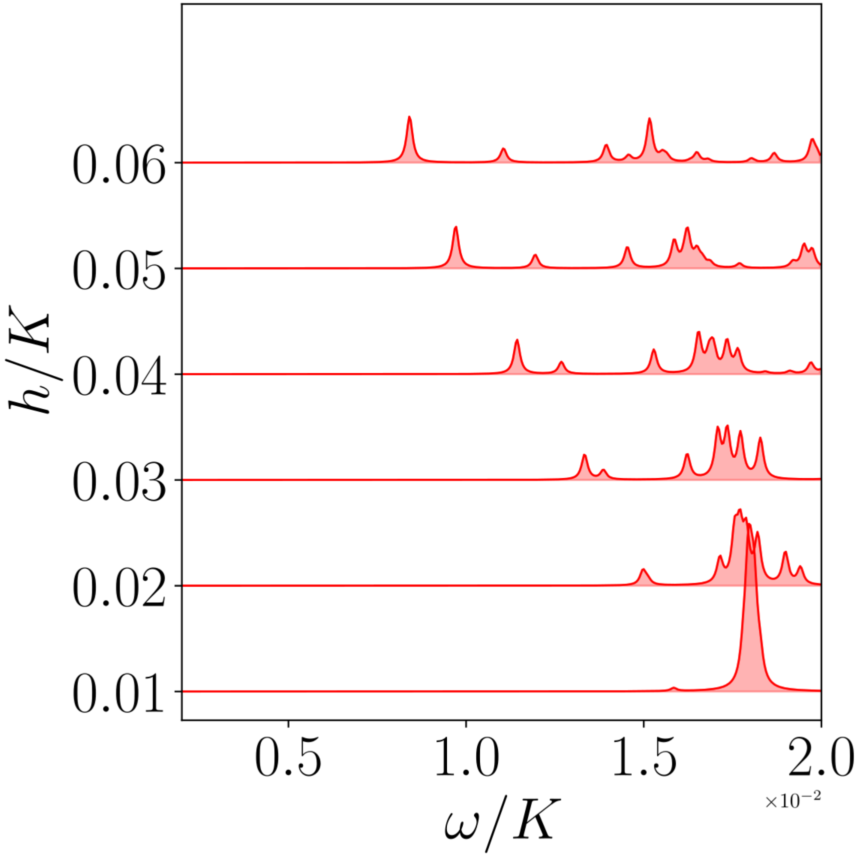
$$H_{\text{eff}} \sim -\sum_s A_s - \sum_p B_p + \sum_i \frac{h^2}{K_z} \tau_i^y$$

Only composite anyon disperses in one-dimensional direction!

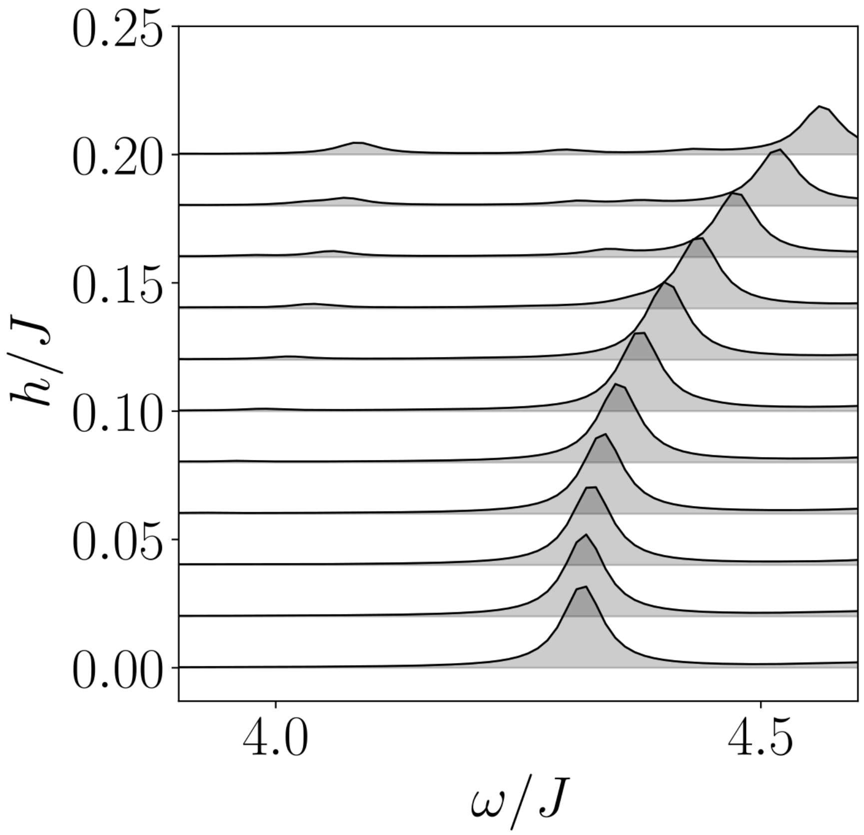
Separation in energy (in inelastic scattering)



Fractionalized particles under field



Normal magnon particles under field



Data obtained by exact diagonalization (ED)

Magnetic-field-induced anyon dynamics

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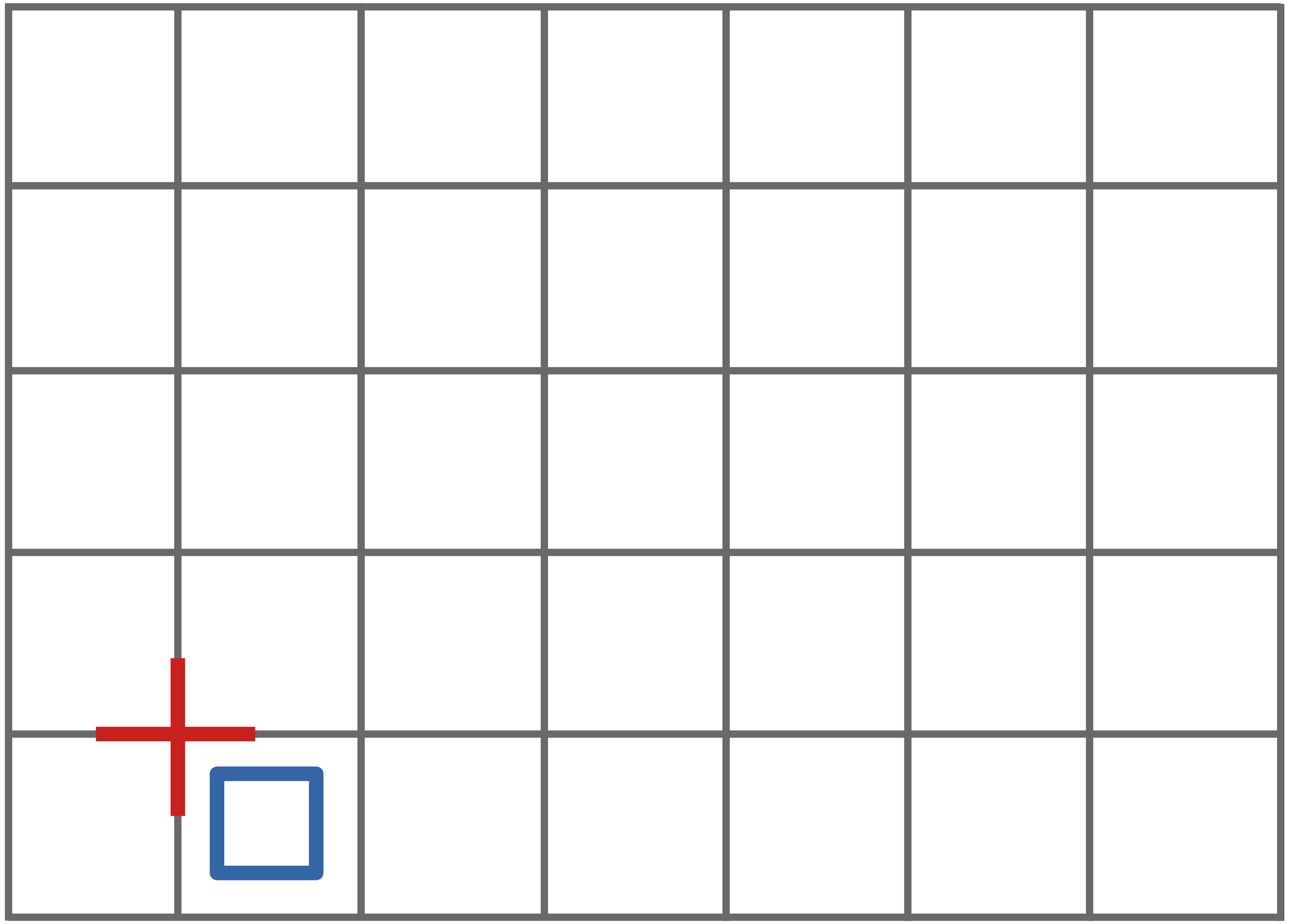

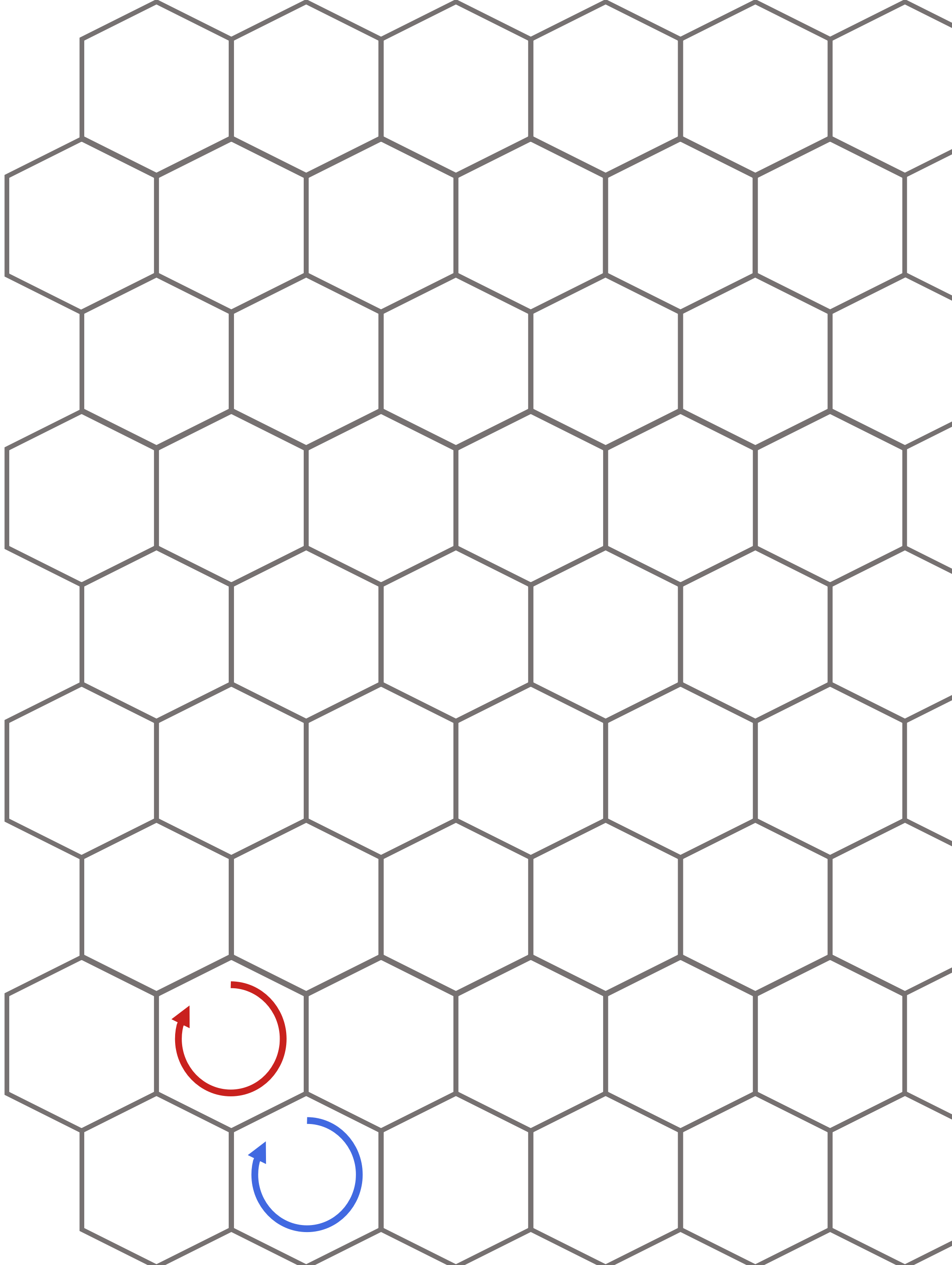

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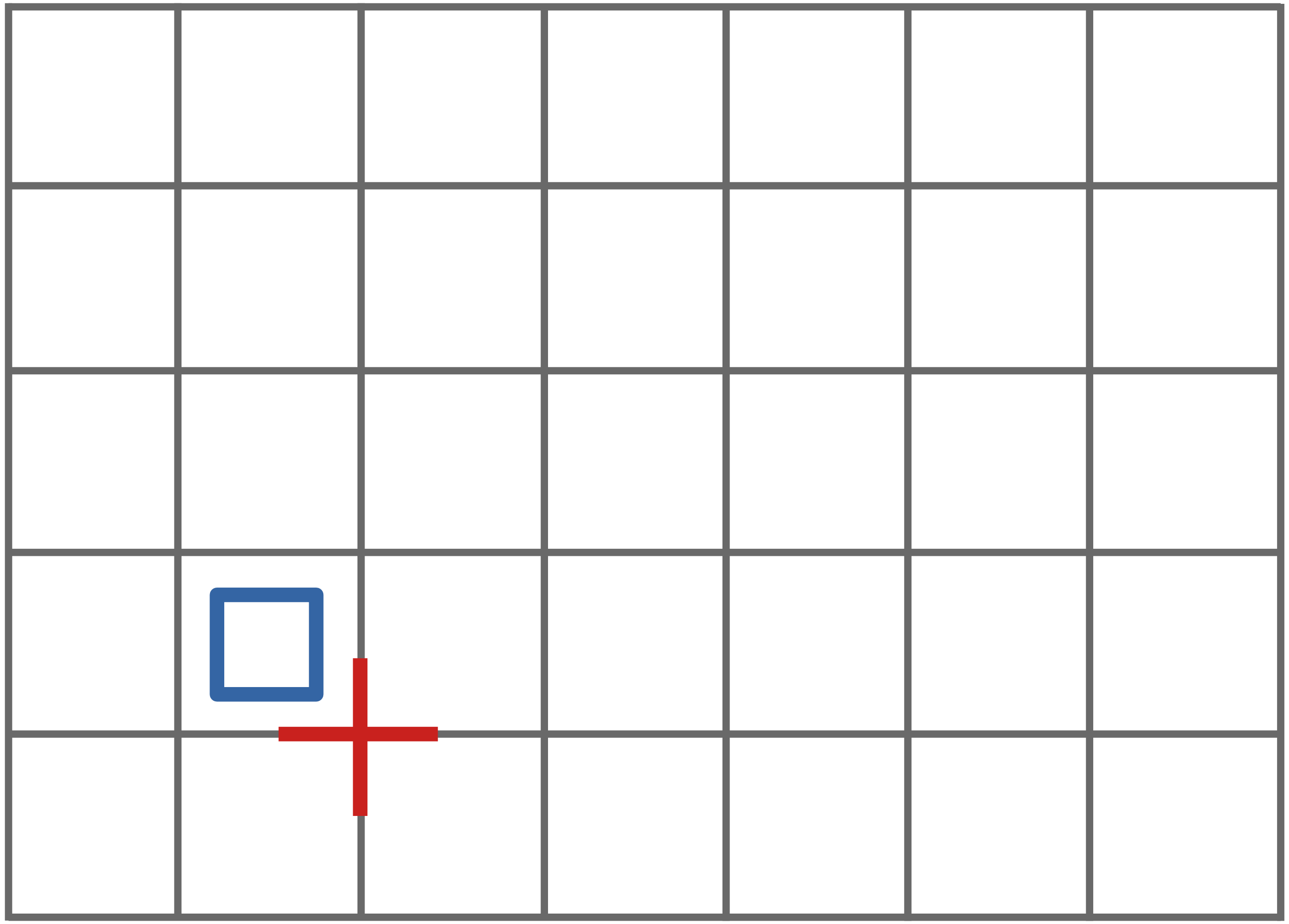

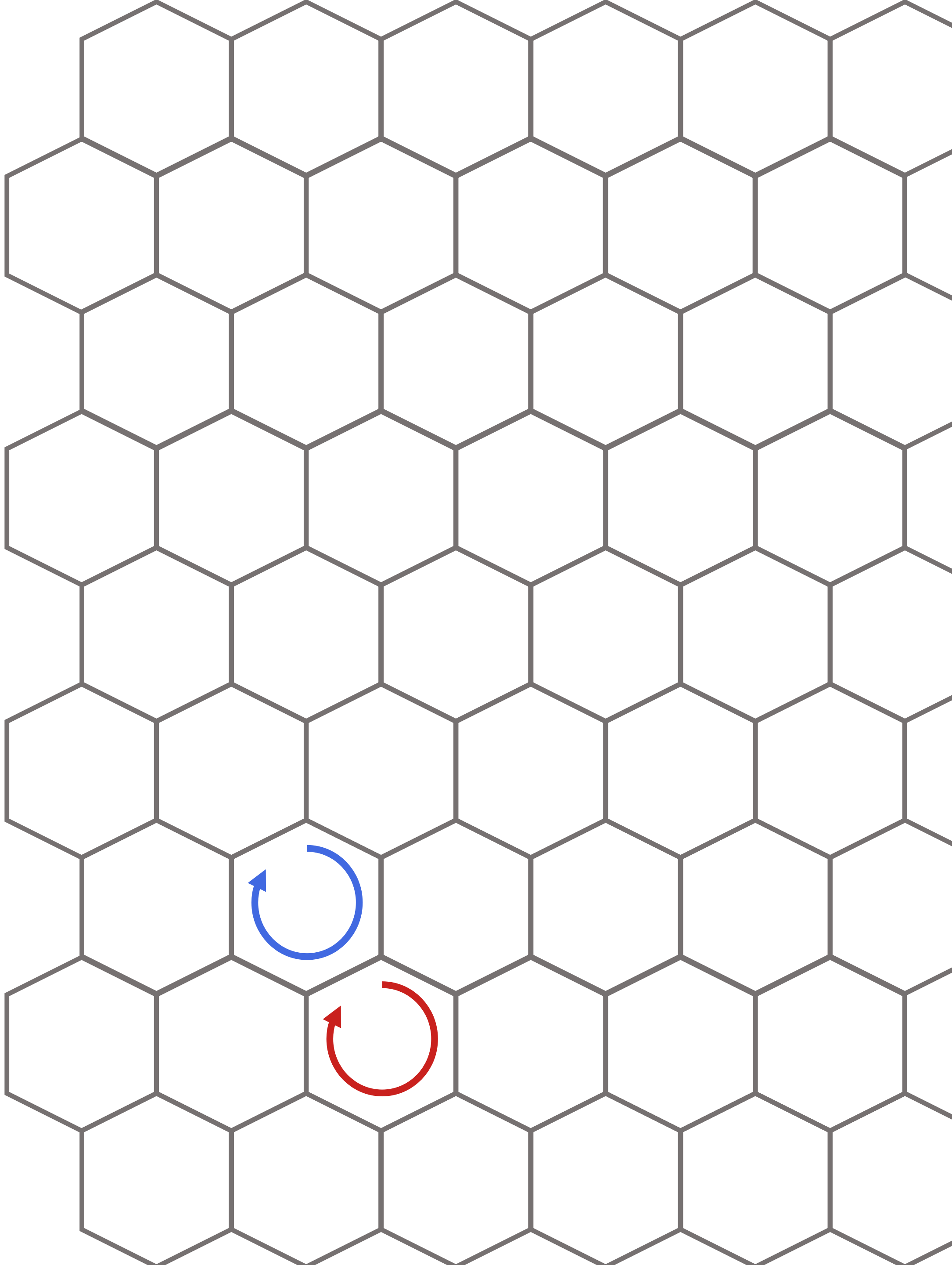

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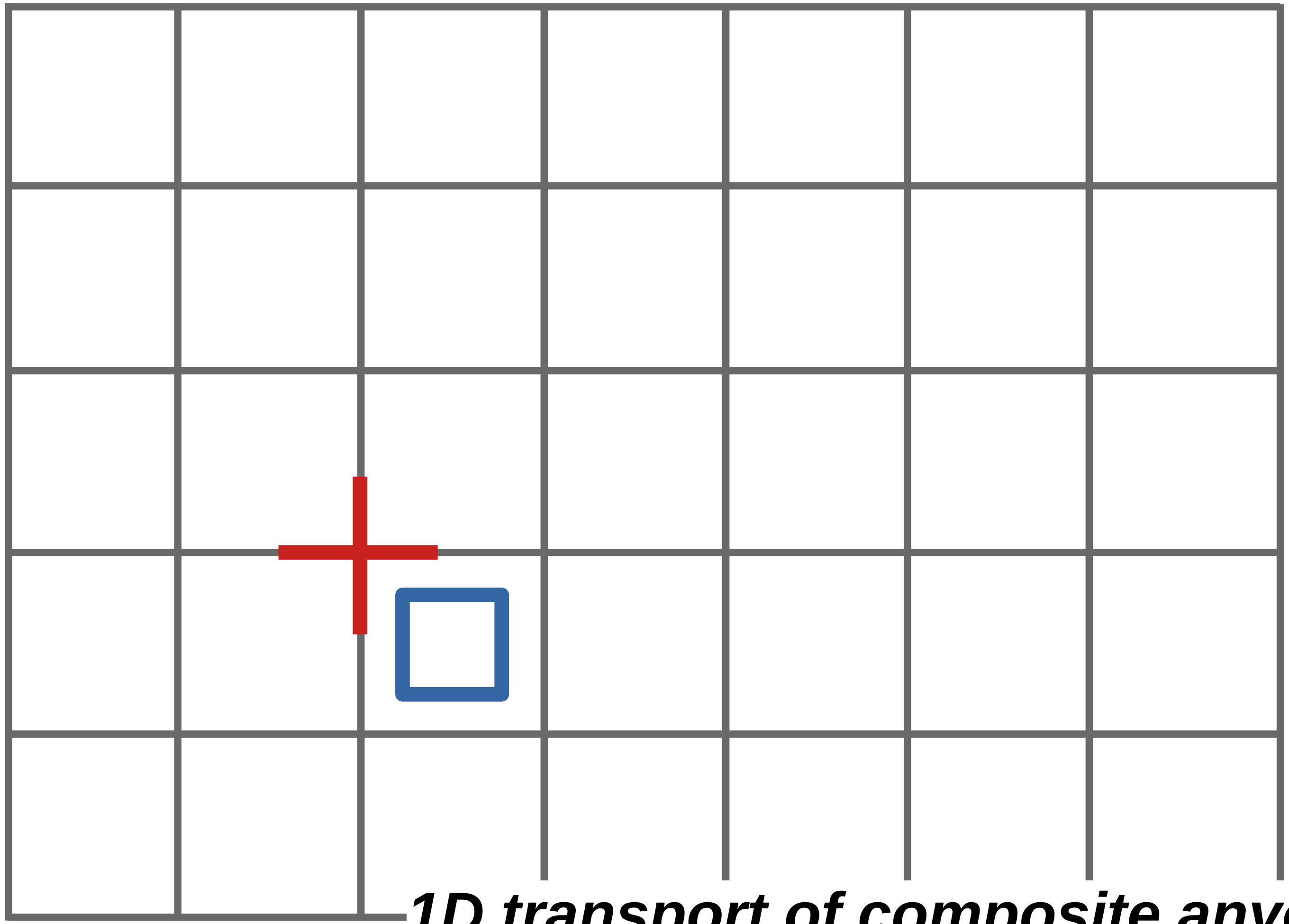

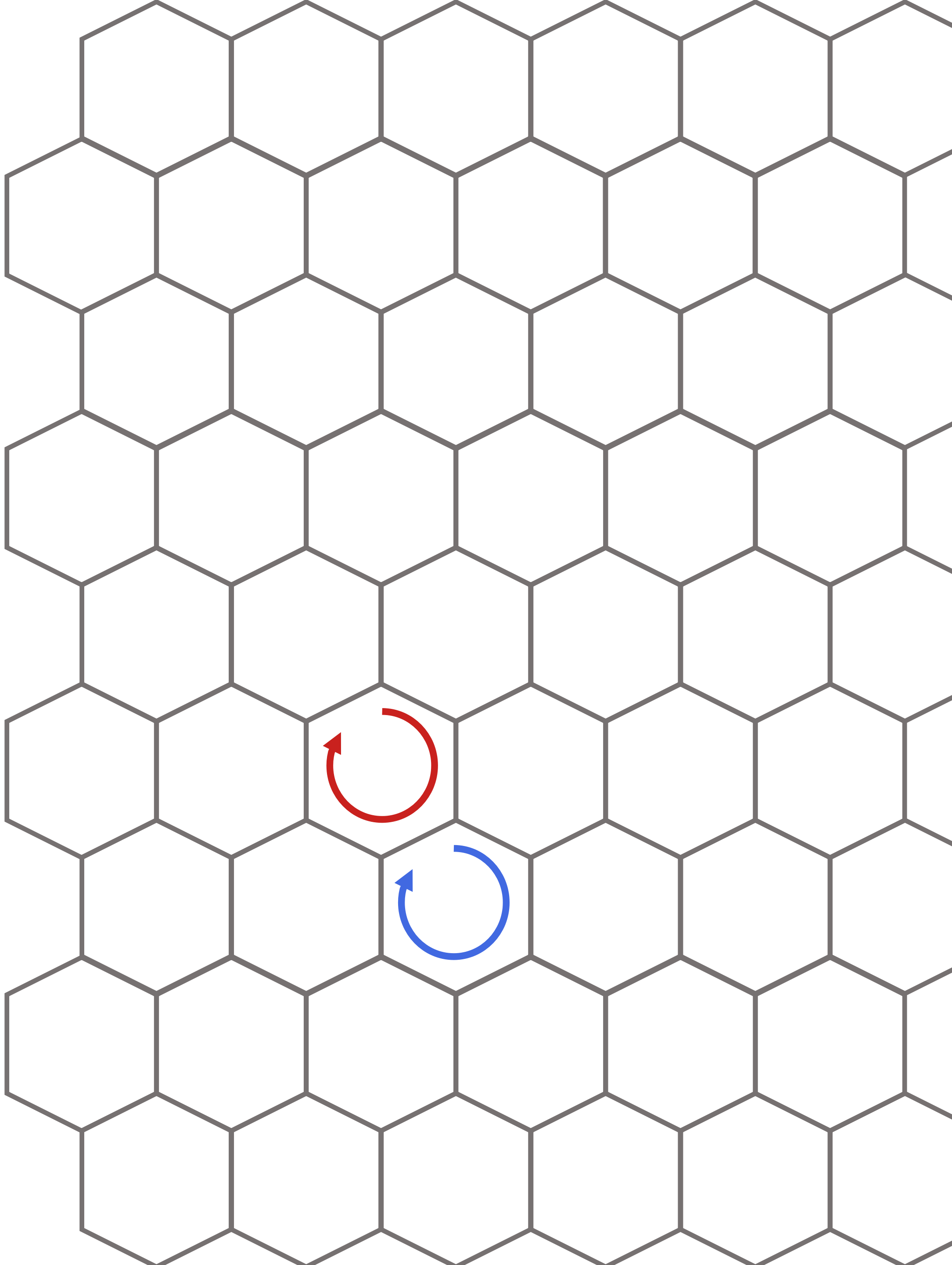

1D transport of composite anyon in a 2D model!

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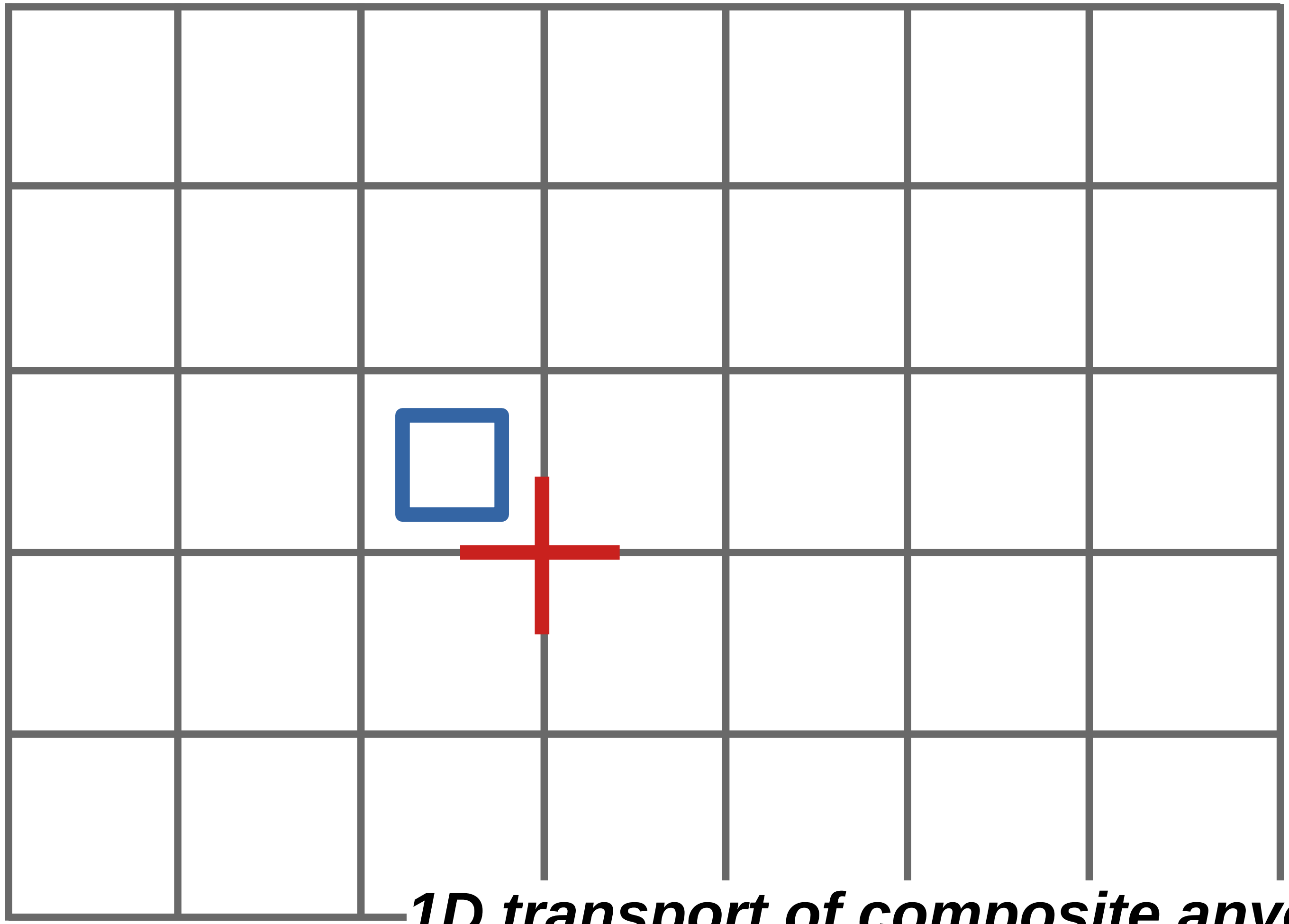

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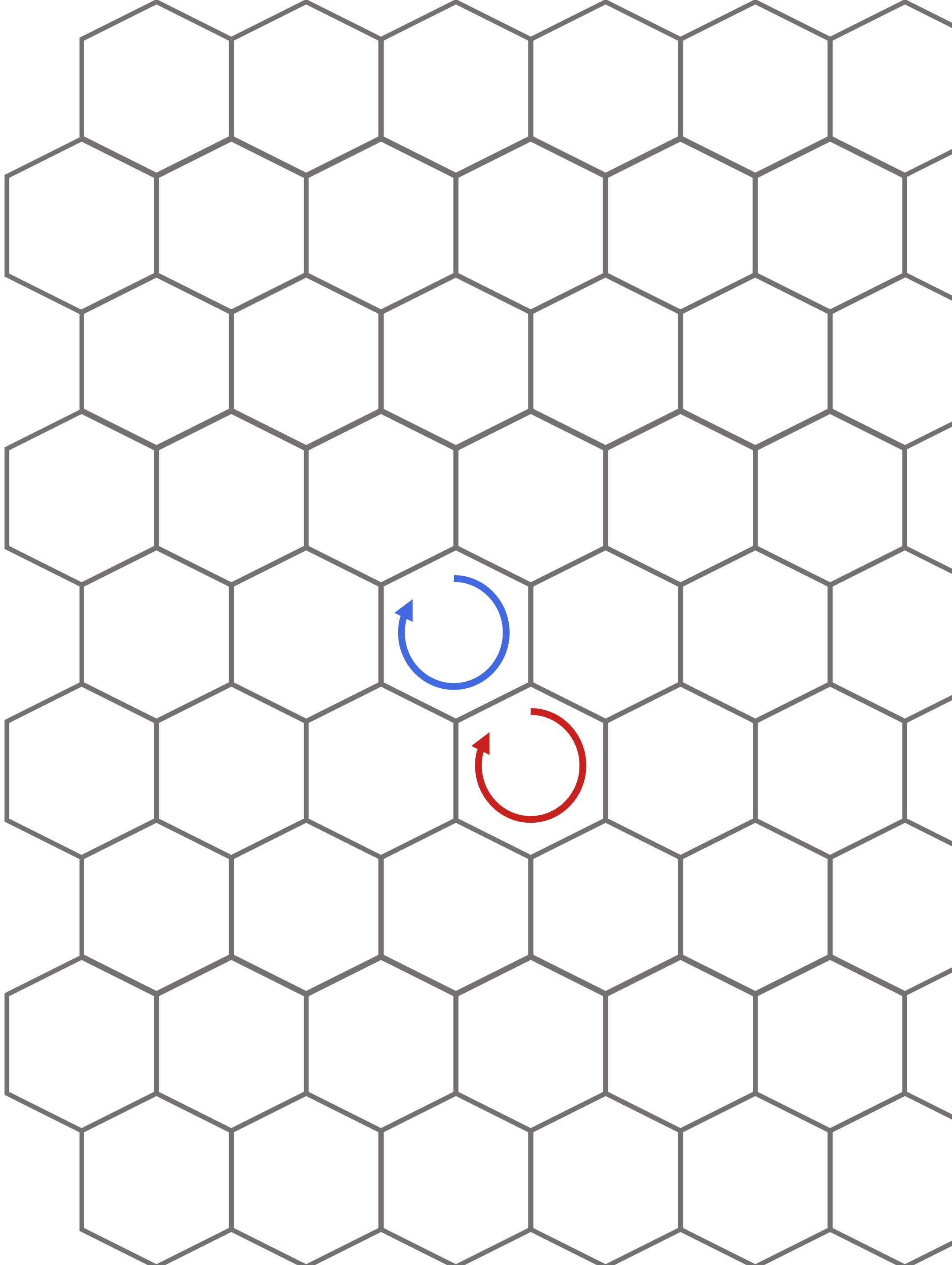

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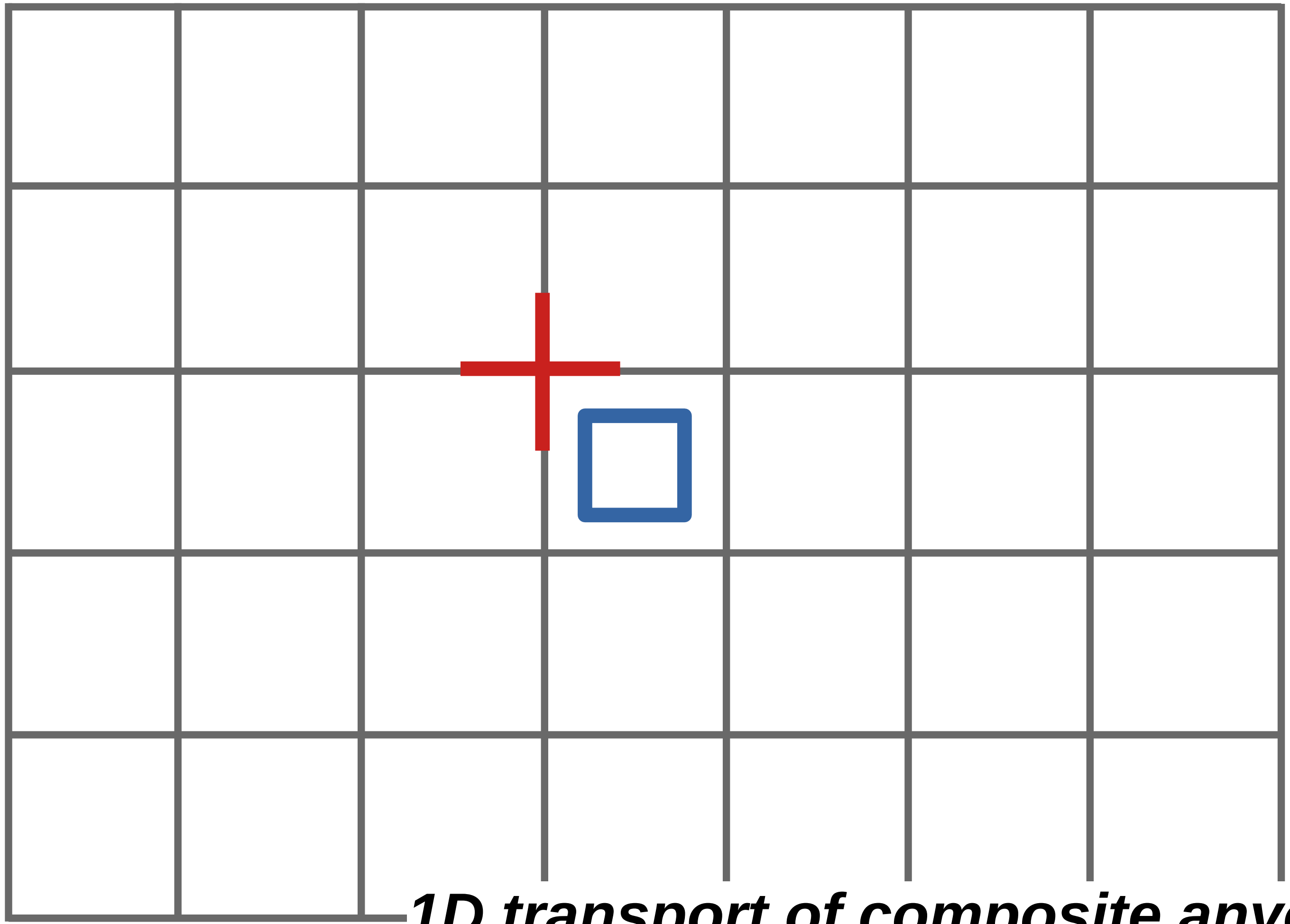

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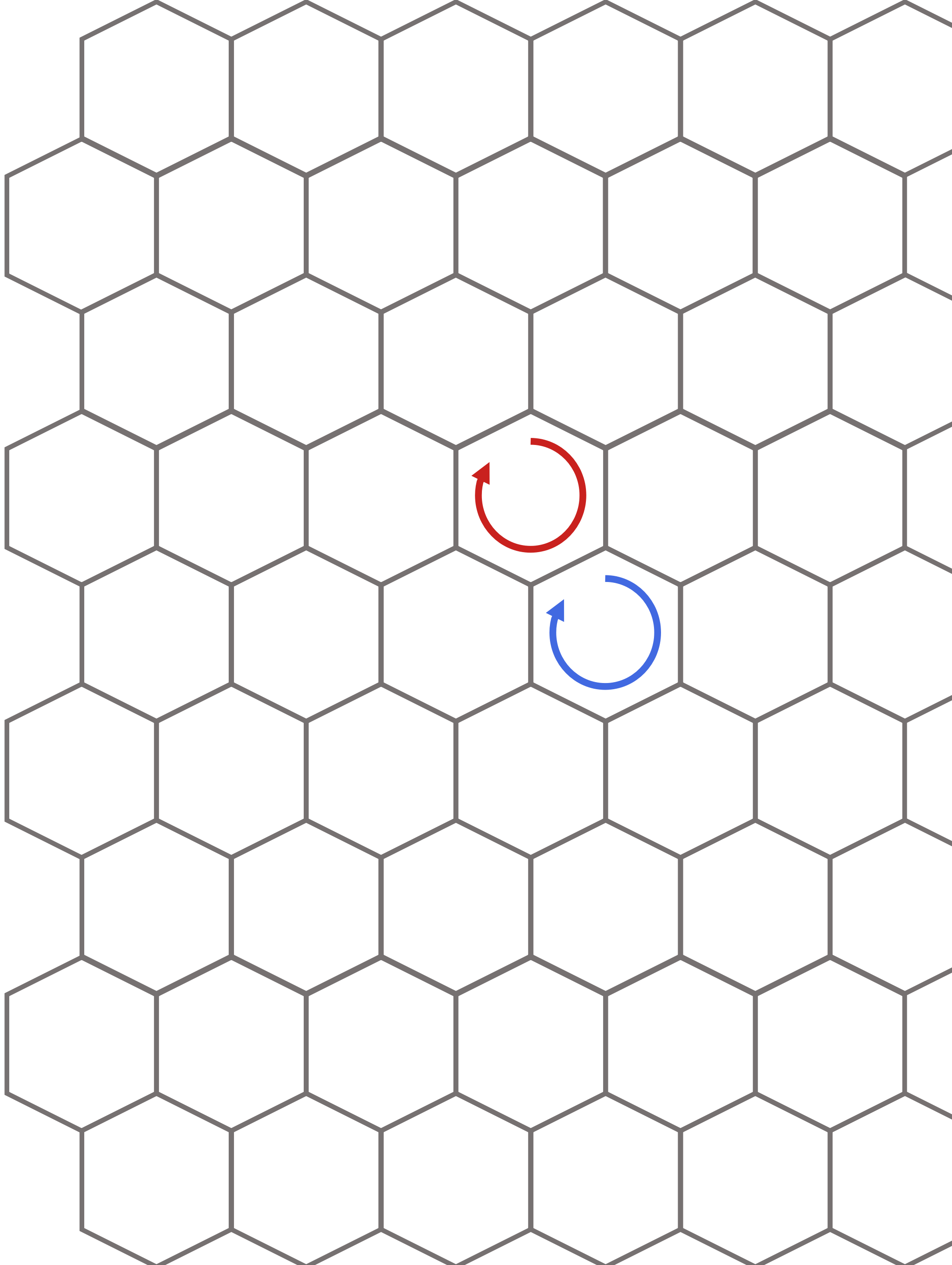

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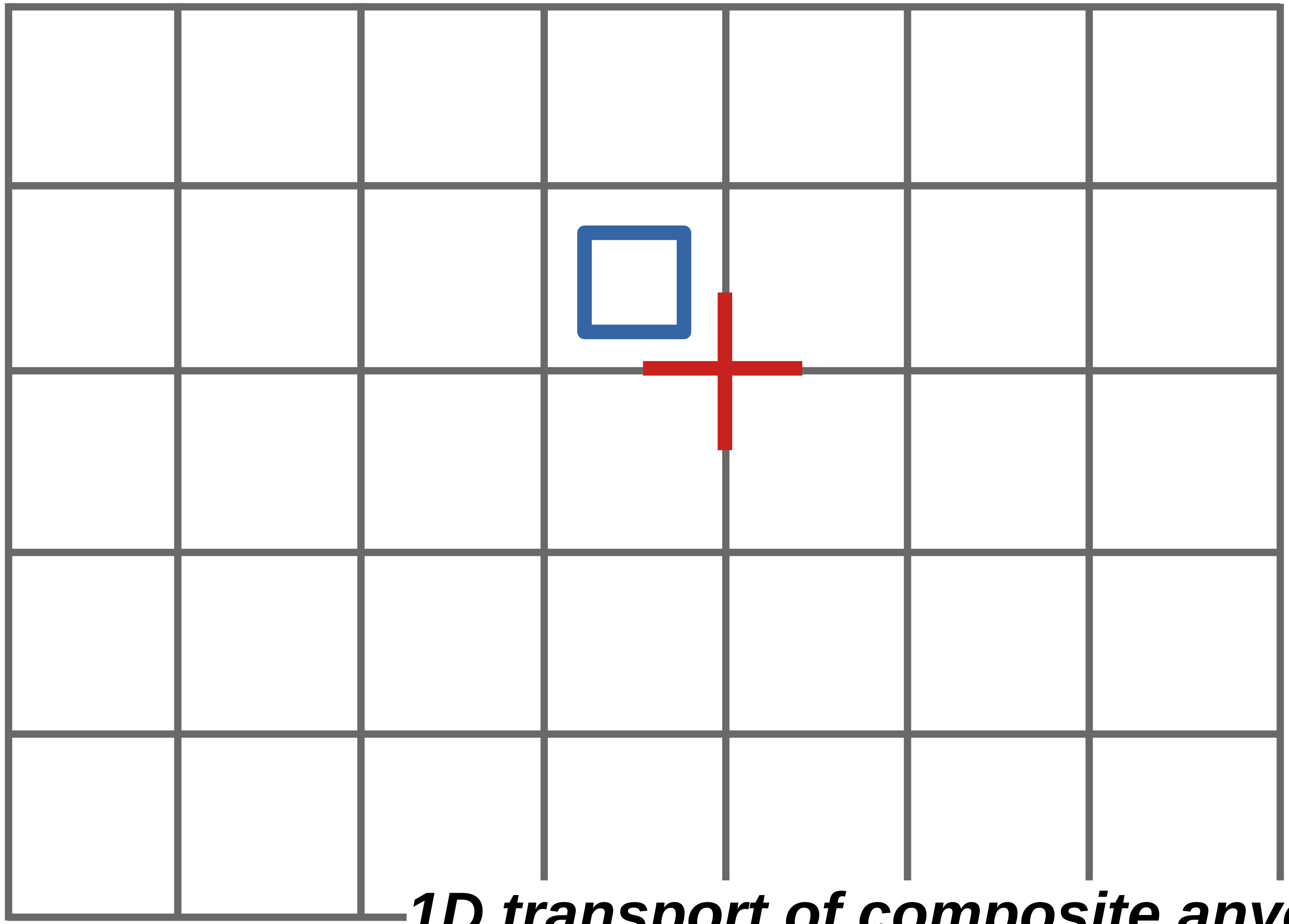

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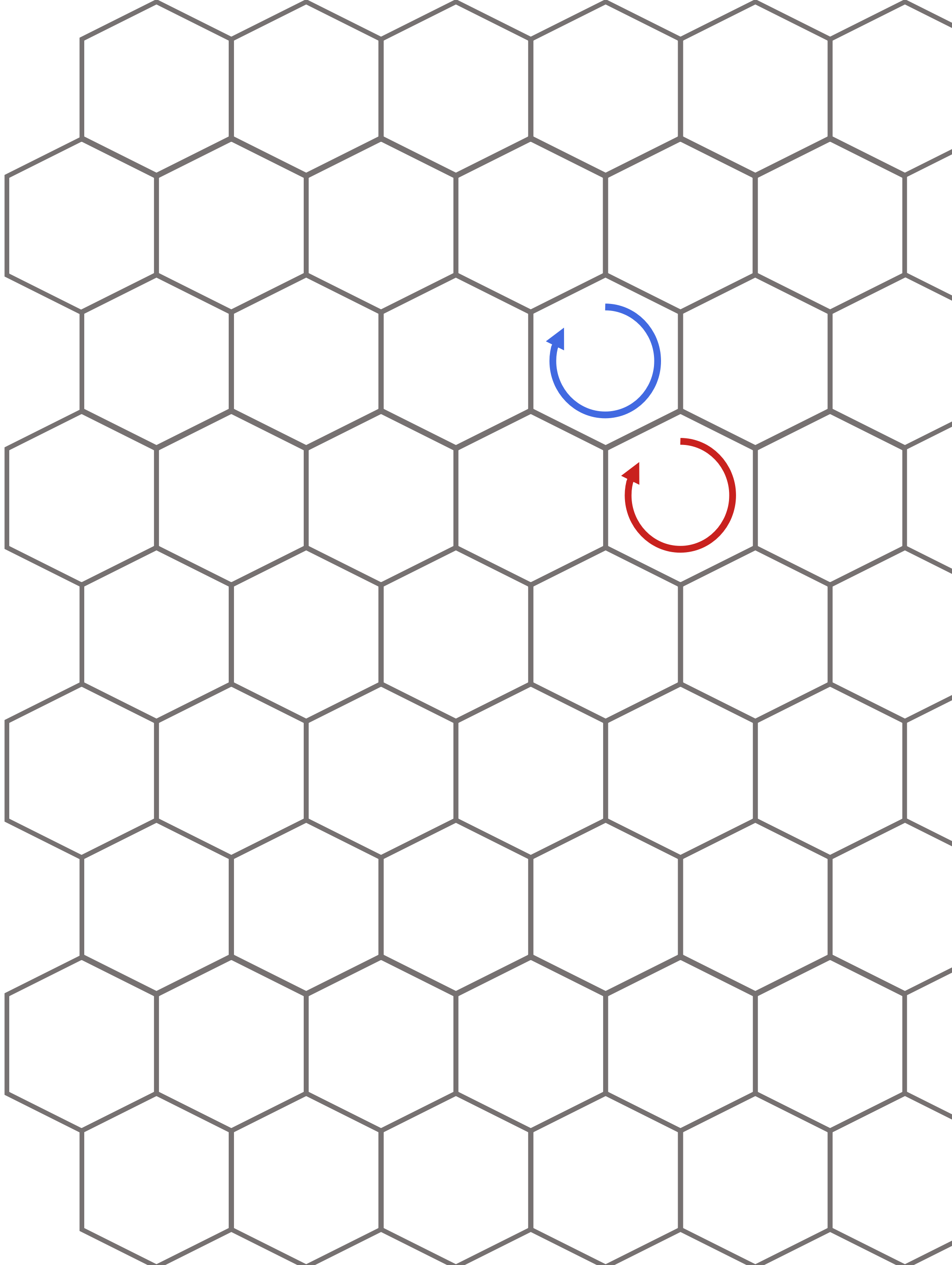

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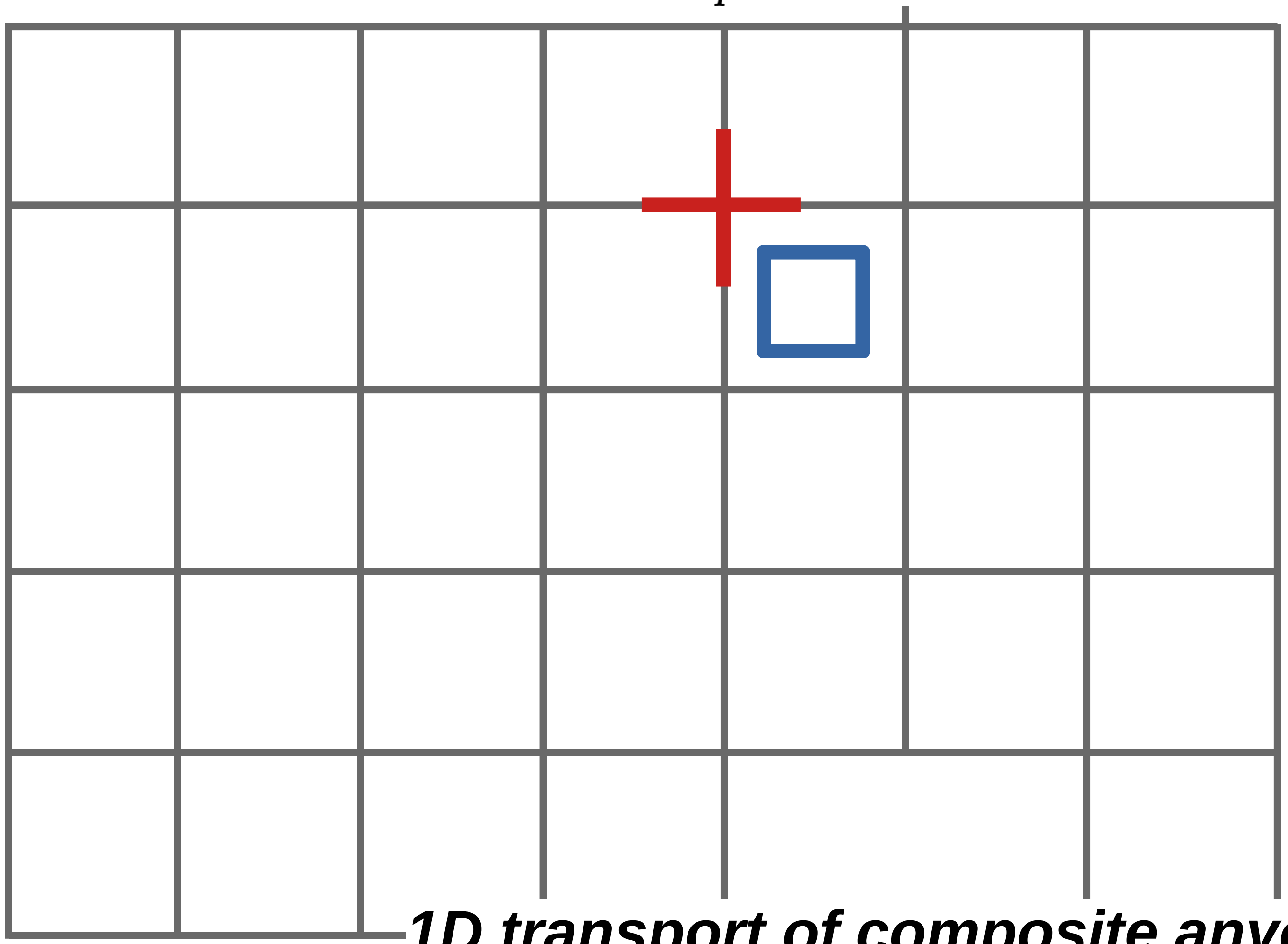

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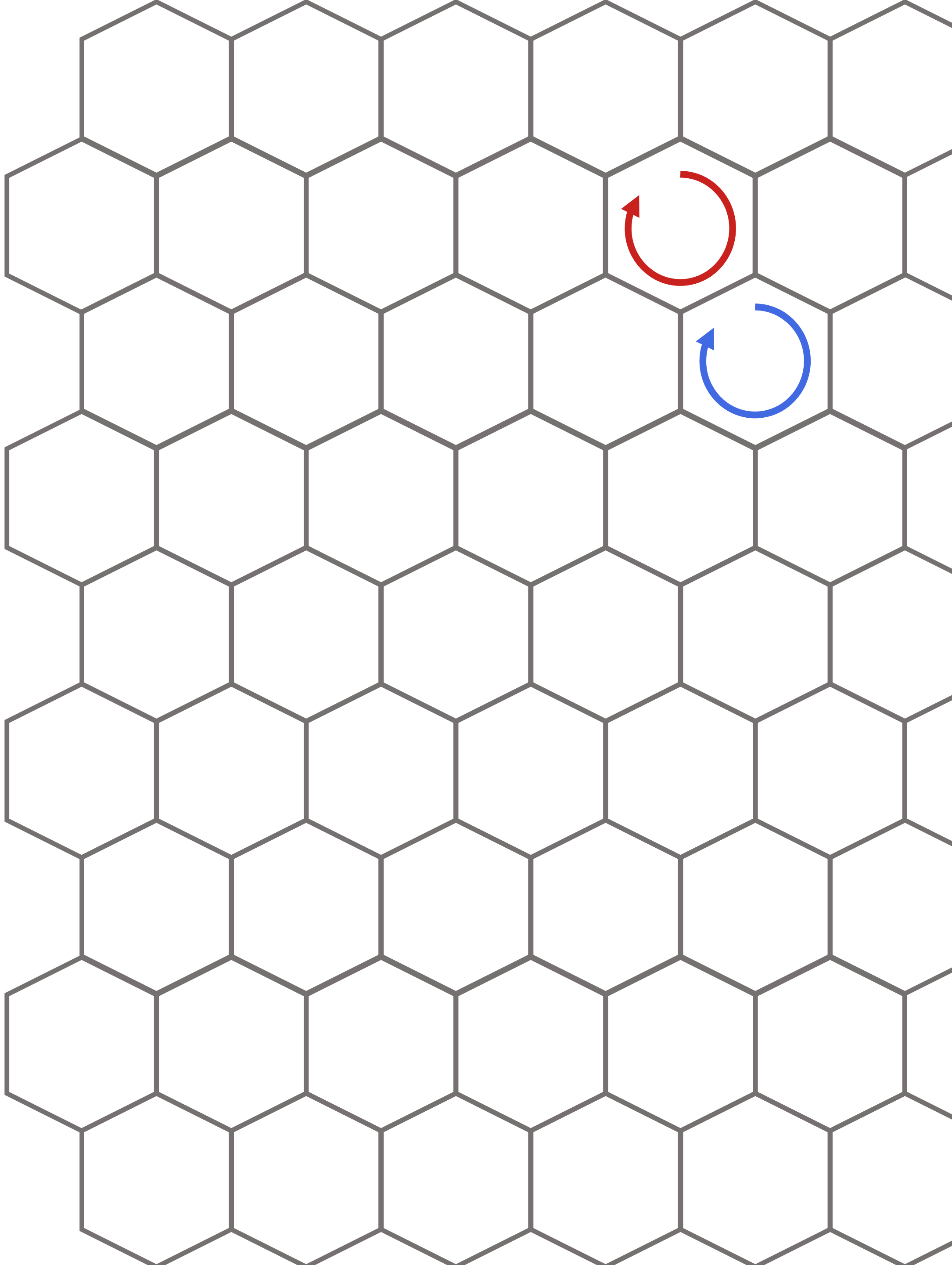

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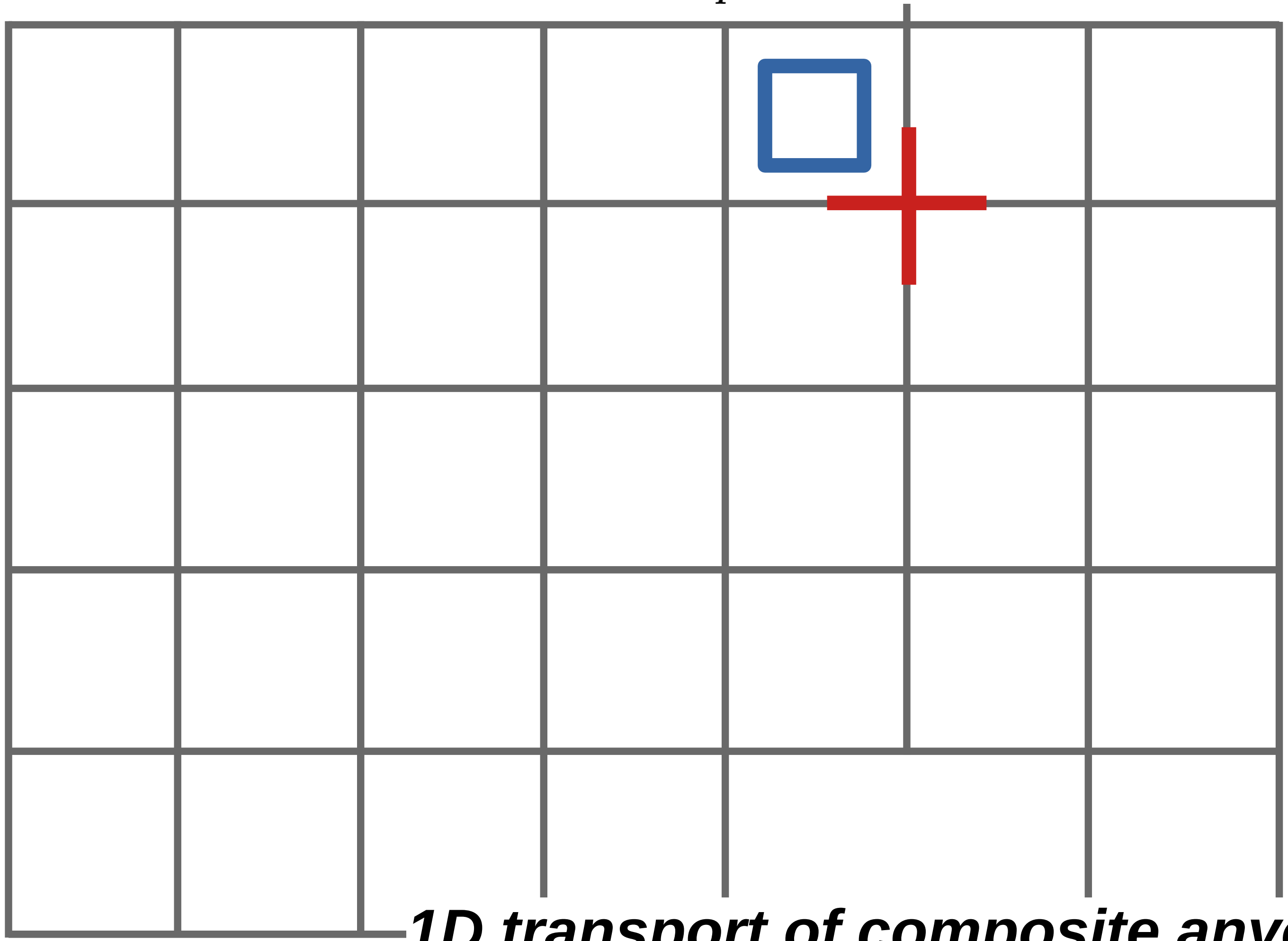

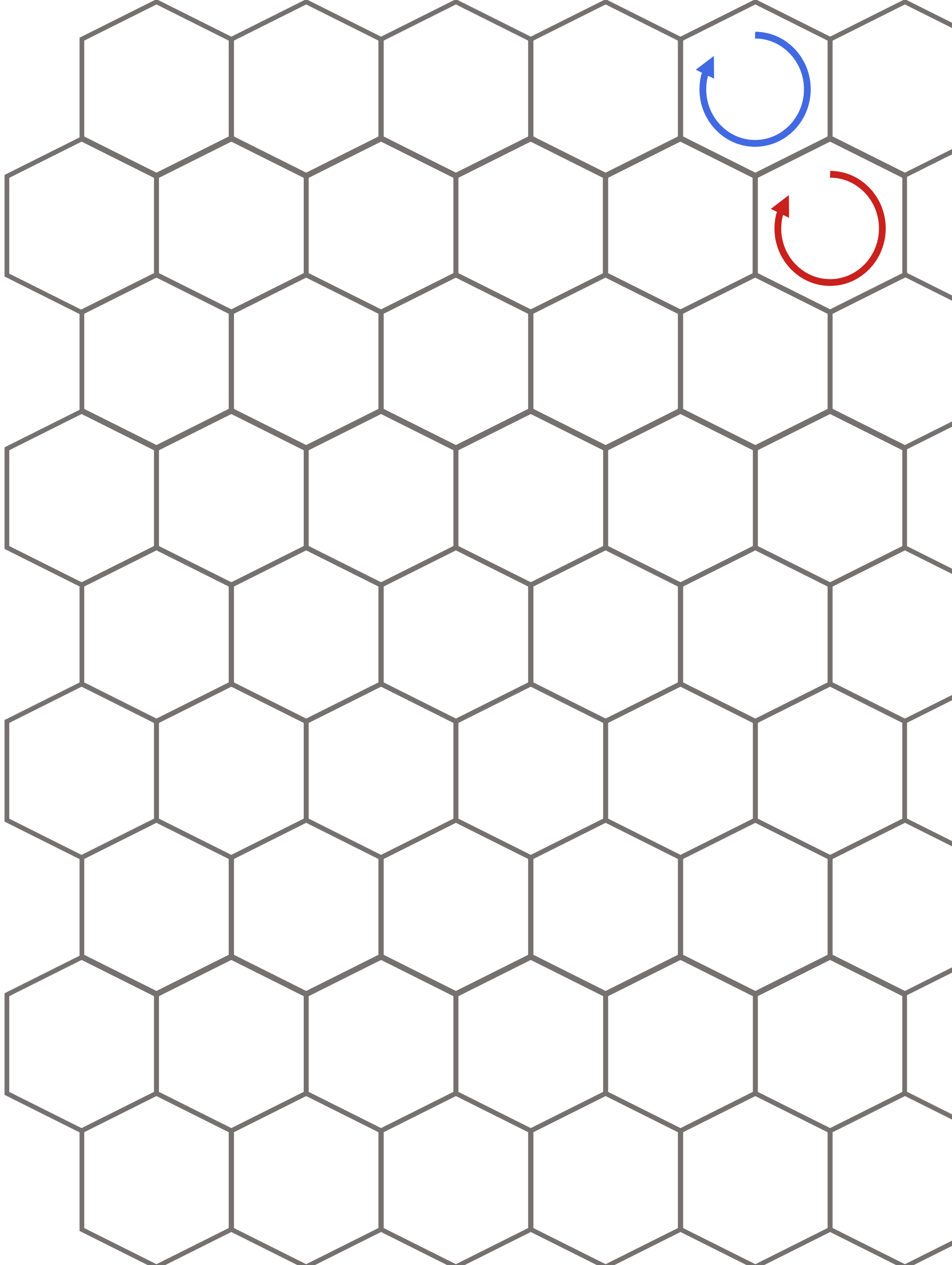

1D transport of composite anyon in a 2D model!

Magnetic-field-induced anyon dynamics

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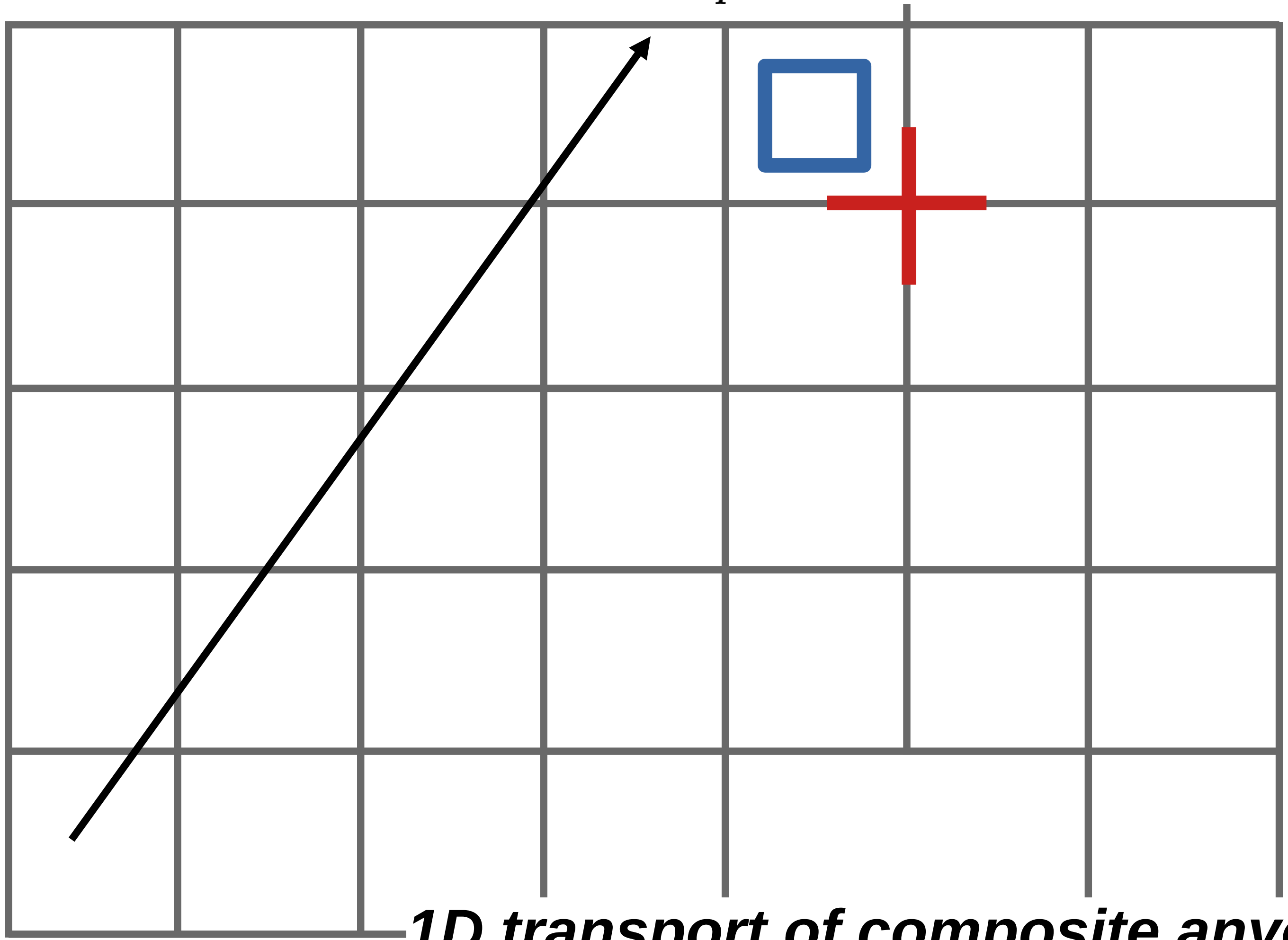
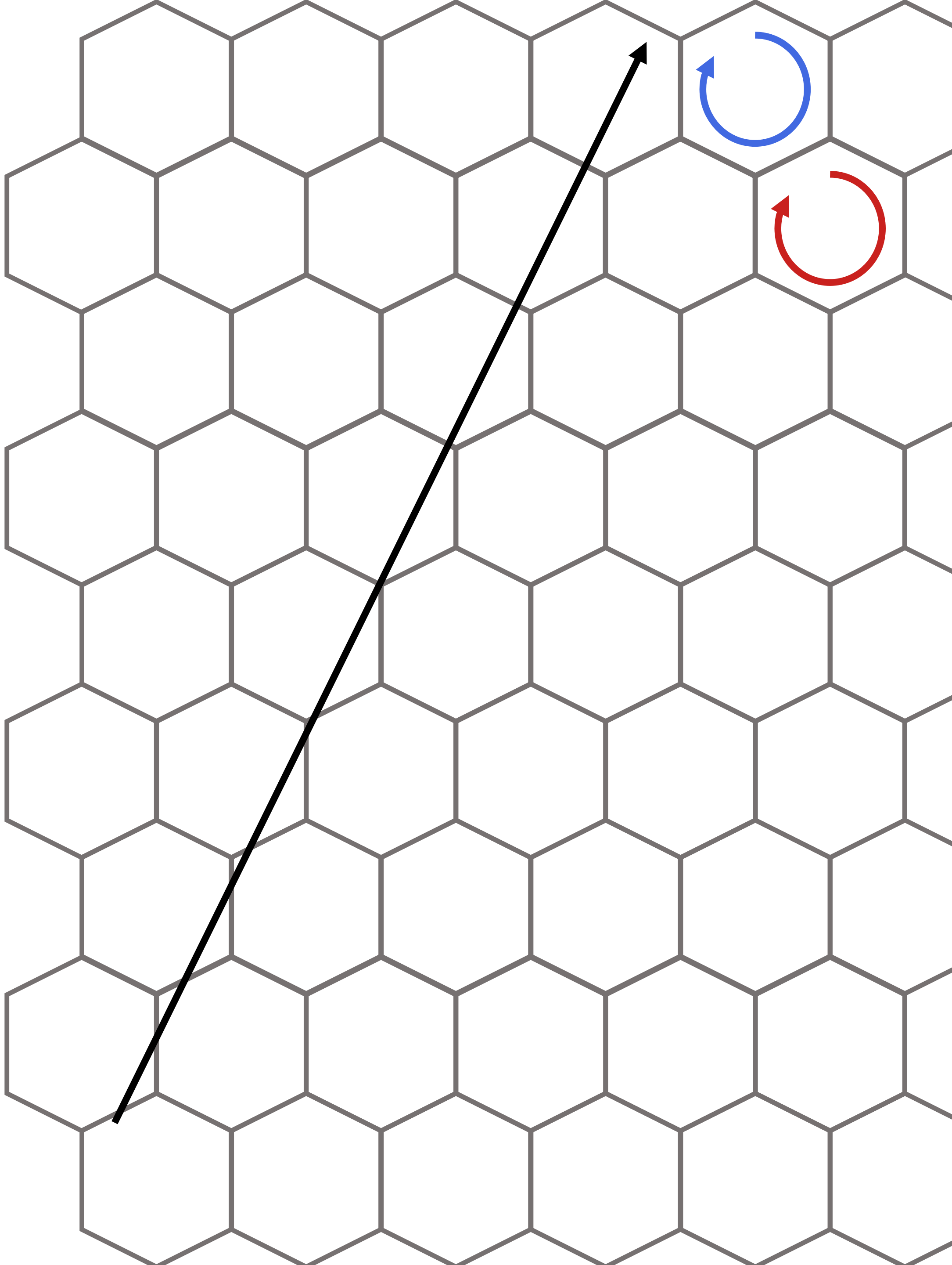
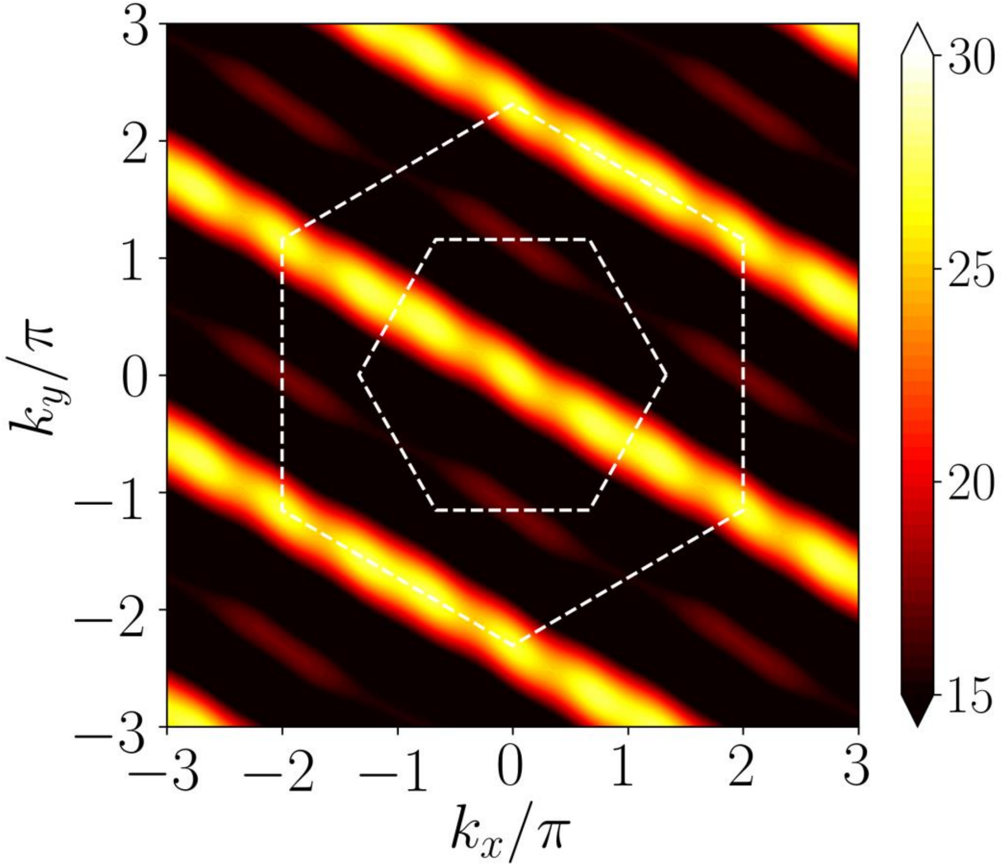




1D transport of composite anyon in a 2D model!

Anyon dispersion in the momentum space



$$\varepsilon(\mathbf{k}) = 4J_{\mathrm{TC}} - \frac{4h^2}{K_z}\cos(\frac{\sqrt{3}}{2}k_x + \frac{3}{2}k_y)$$

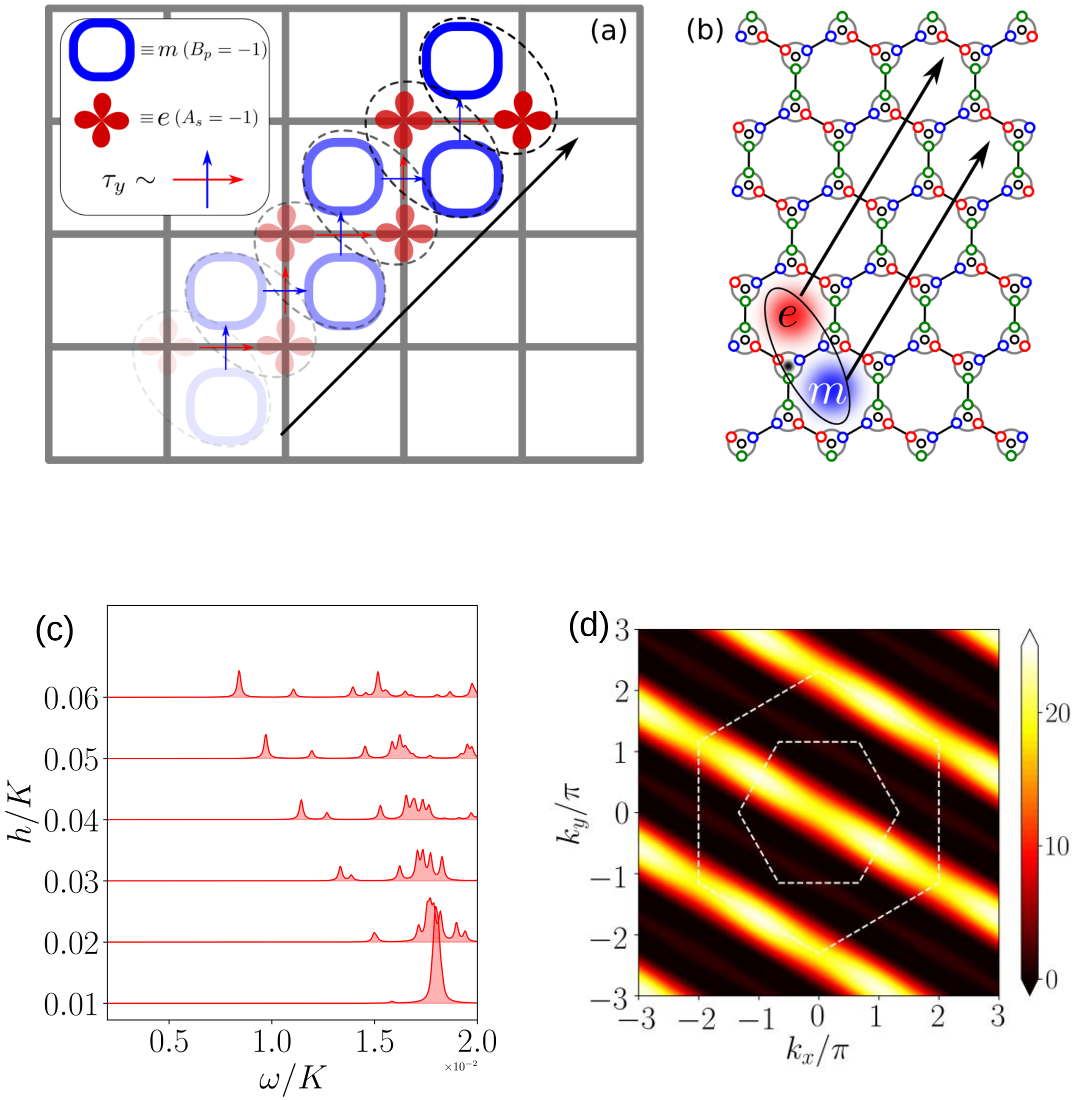
$$\mathbf{v}(\mathbf{k}) = \partial_{\mathbf{k}}\varepsilon_x(\mathbf{k}) = \frac{4h^2}{K_z}\mathbf{d}_1\sin(\mathbf{d}_1\cdot\mathbf{k})$$

Velocity in $\mathbf{d}_1 = (\frac{\sqrt{3}}{2}, \frac{3}{2})$ **direction**

Data obtained by dynamical density matrix renormalization group (DMRG)

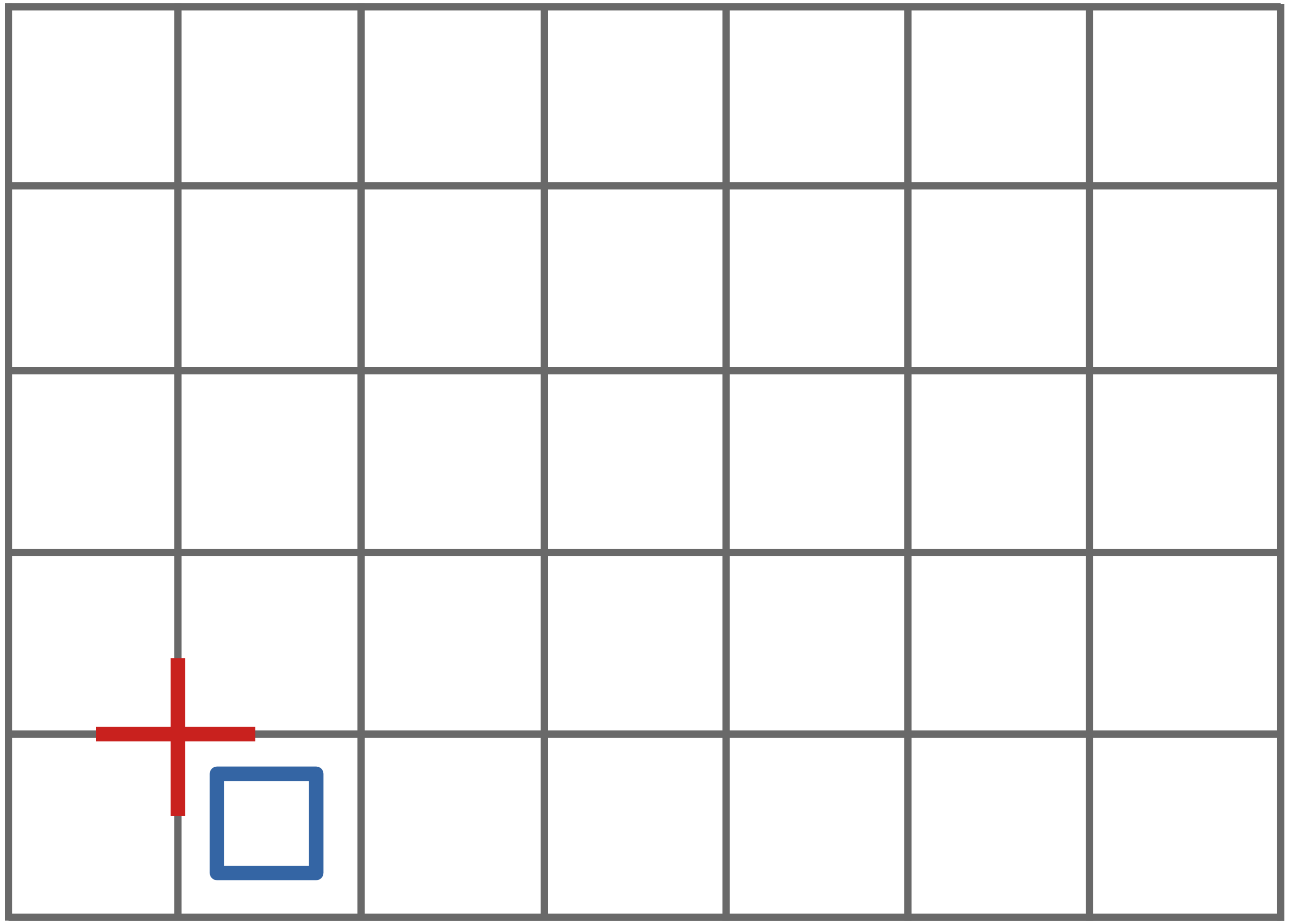
Summary

- In Kitaev quantum spin liquid, each **spin-1/2 fractionalizes into anyons**
- In high anisotropy limit, anyons become low- energy excitations
- **Under an out-of-plane magnetic field, only the composite anyon propogate**
- **The field-induced composite anyons disperse in 1D (*Perturbation theory*)**
- **The composite anyons exhibit sharp, definitive signal in inelastic scattering (ED, DMRG)**
- Implying possible probes of anyon by inelastic light or neutron scattering experiments



Magnetic-field-induced anyon dynamics

$$H_{\mathrm{eff}} \sim -\sum_s A_s - \sum_p B_p + \sum_i \frac{h^2}{K_z} \tau_i^y$$



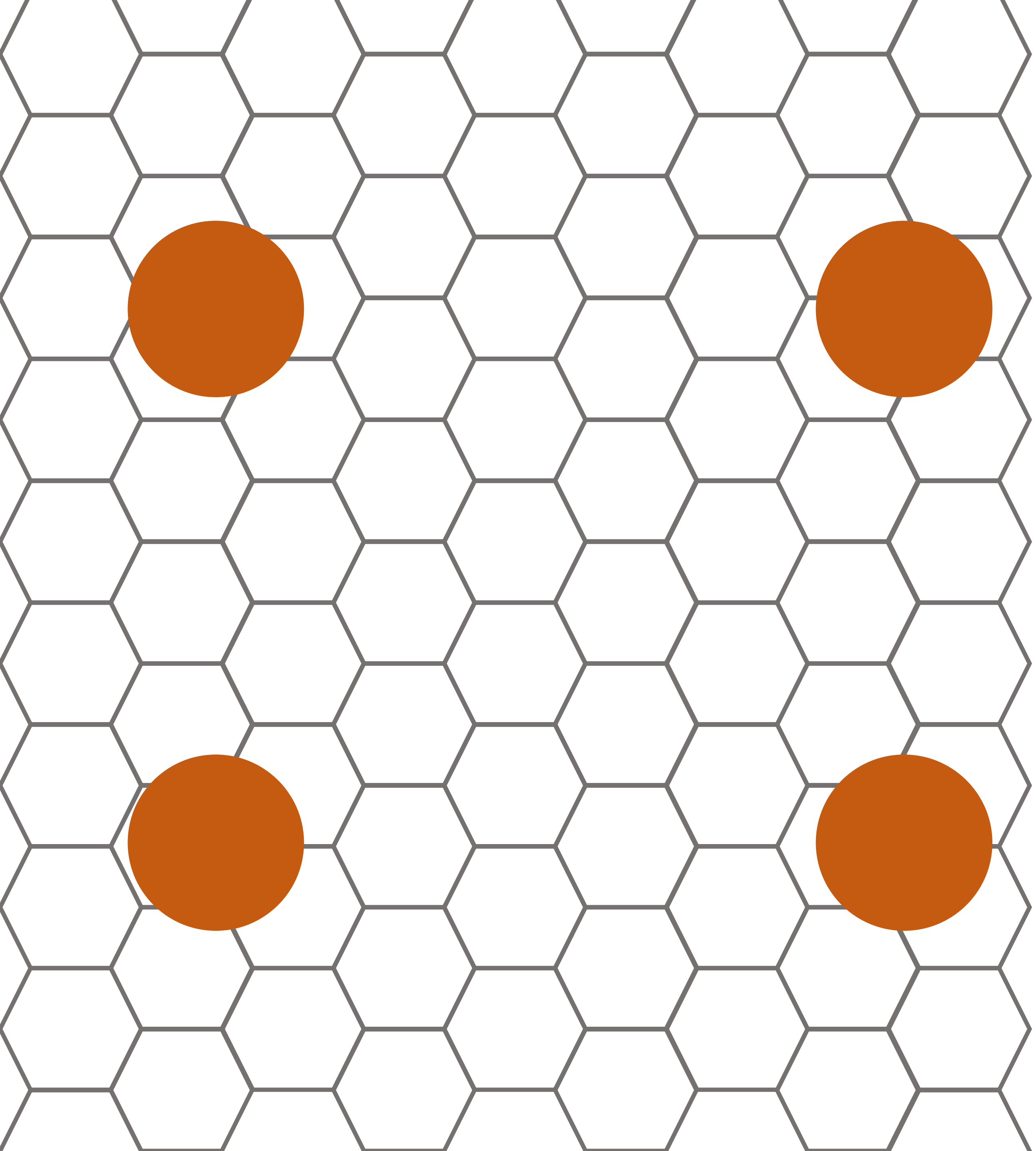
$$A_s = \prod_{i \in s} \tau_i^x$$

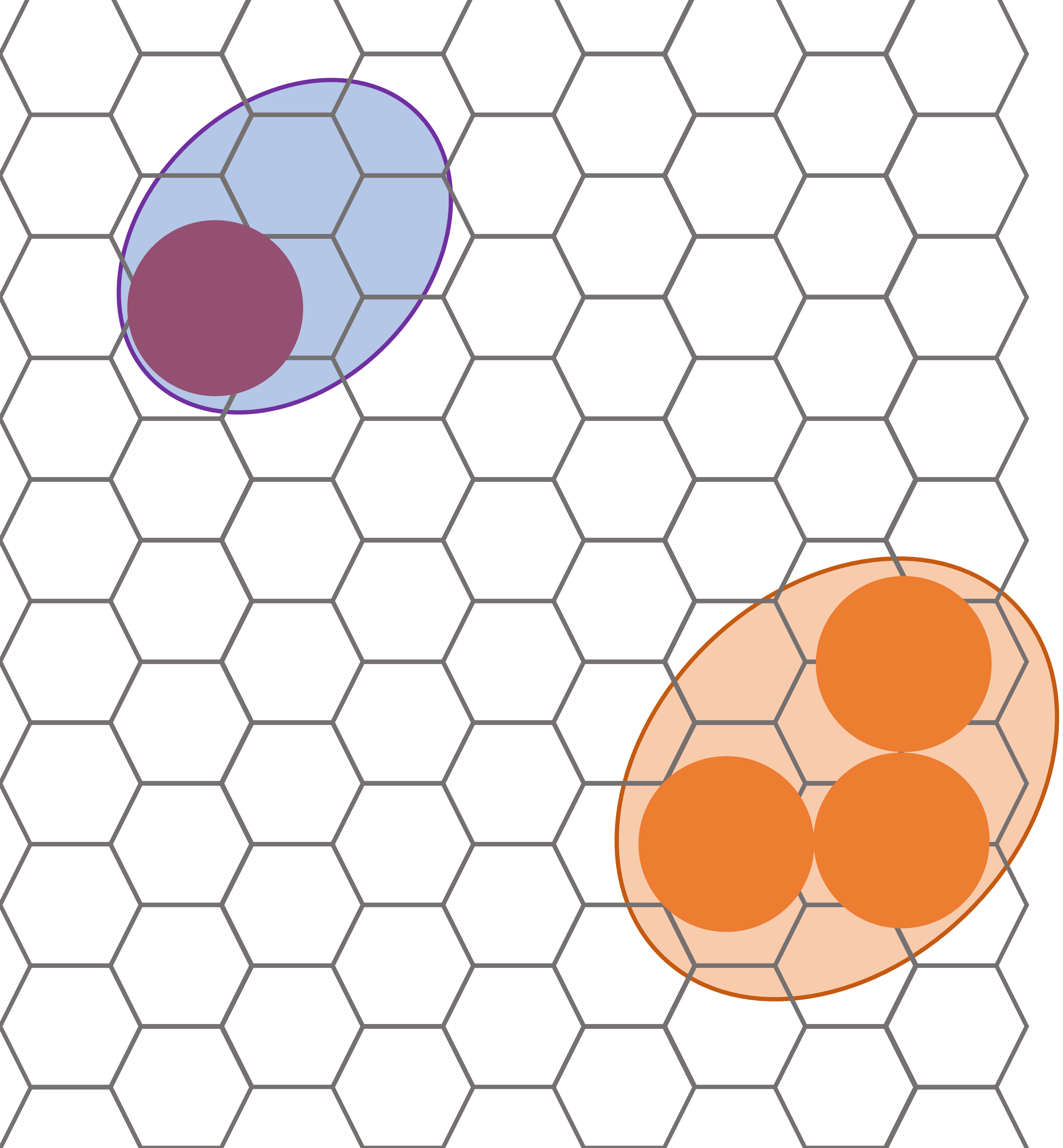
$$B_p = \prod_{i \in p} \tau_i^z$$

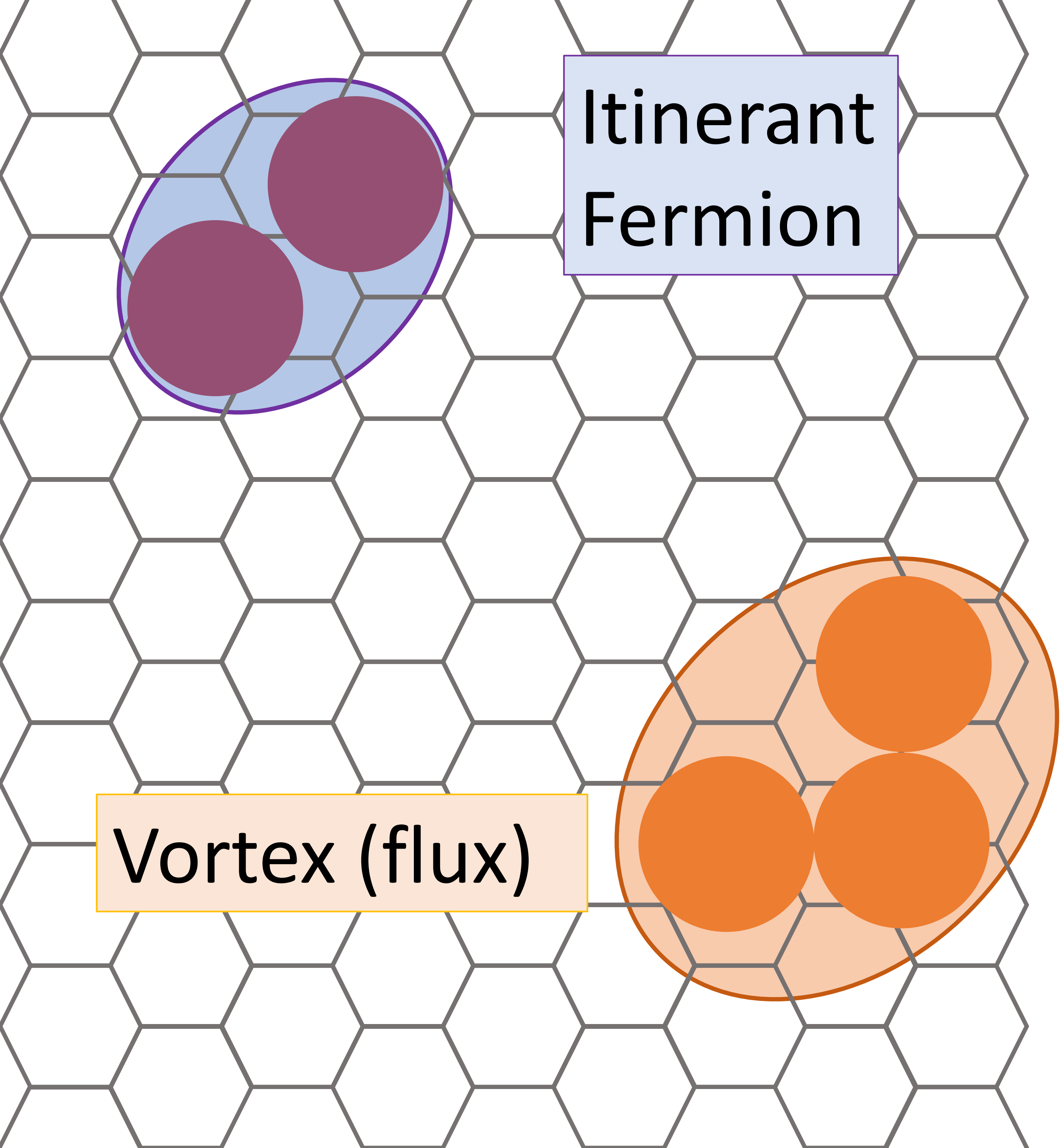
$$\tau_i^y \sim \tau_i^z \tau_i^x$$

$$[\tau_i^x, A_s] = 0$$

$$[\tau_i^z, B_p] = 0$$






Itinerant
Fermion
Vortex (flux)