

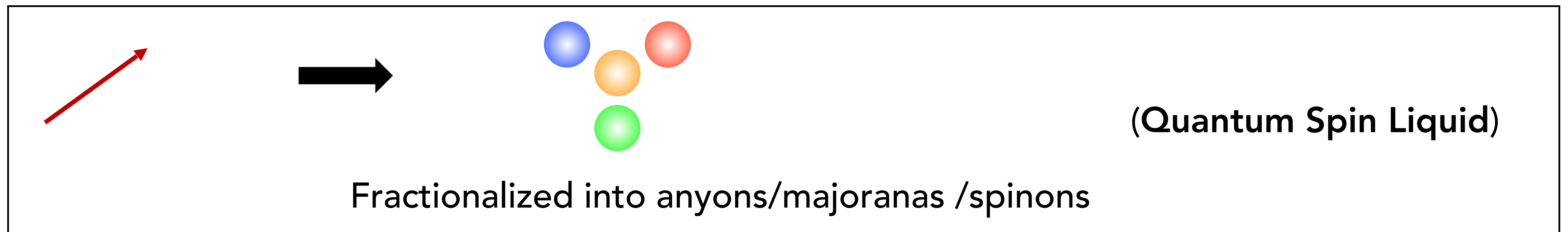
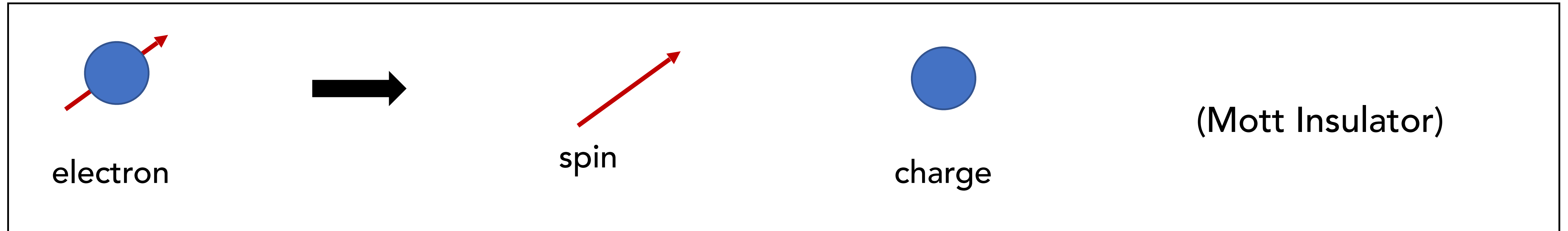


# Constraint mobility of fractionalized particles in quantum spin liquid

Shi Feng, Nandini Trivedi  
Department of Physics, The Ohio State University

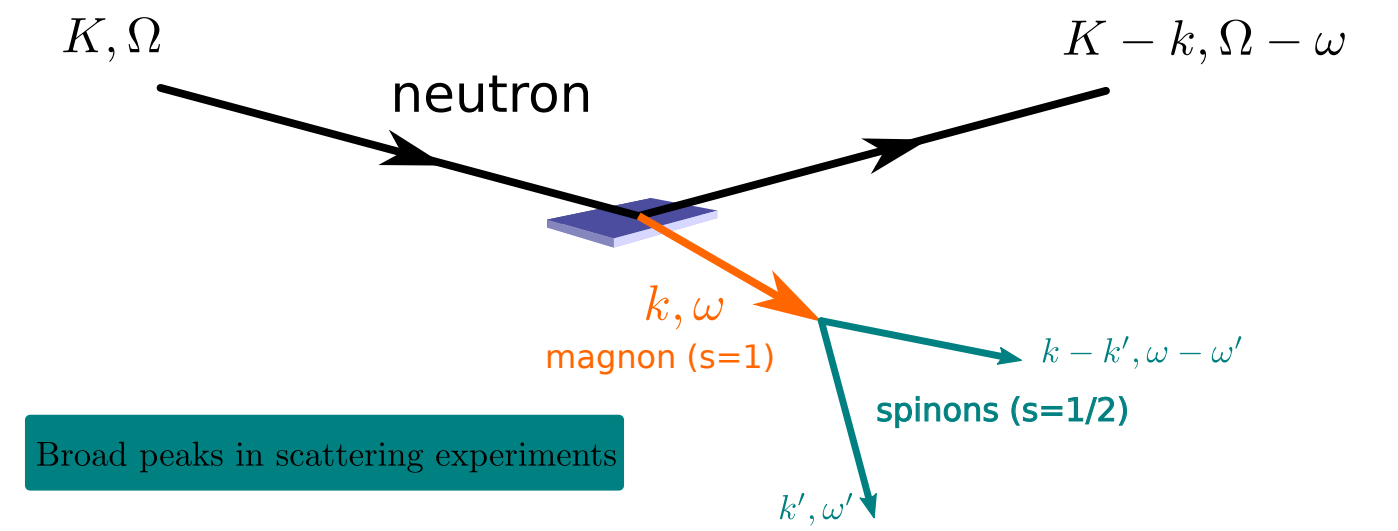
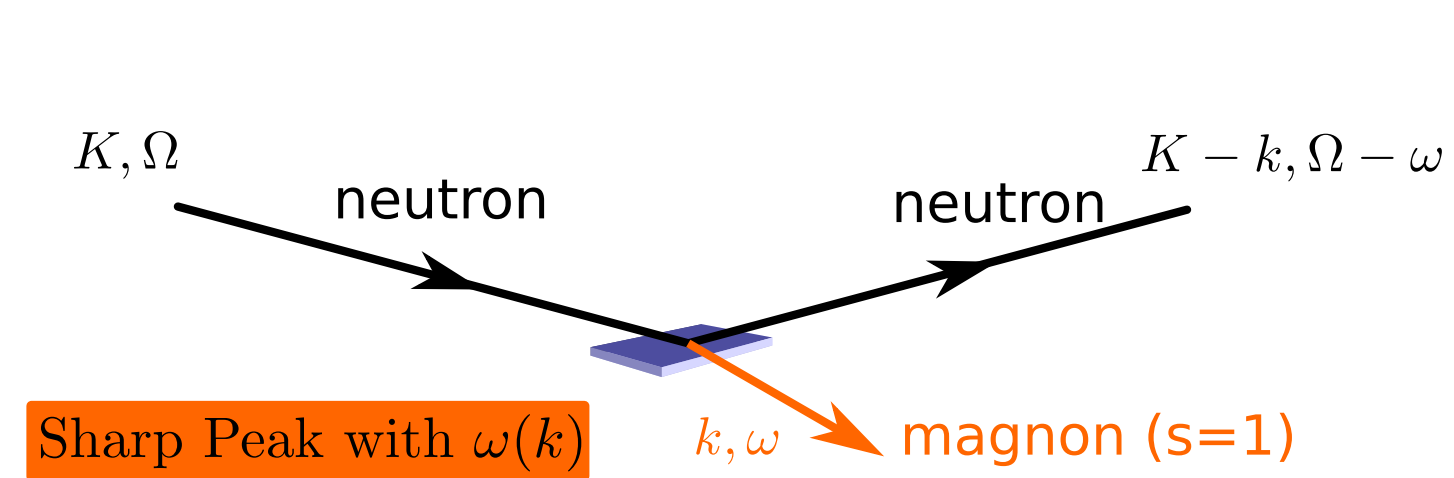
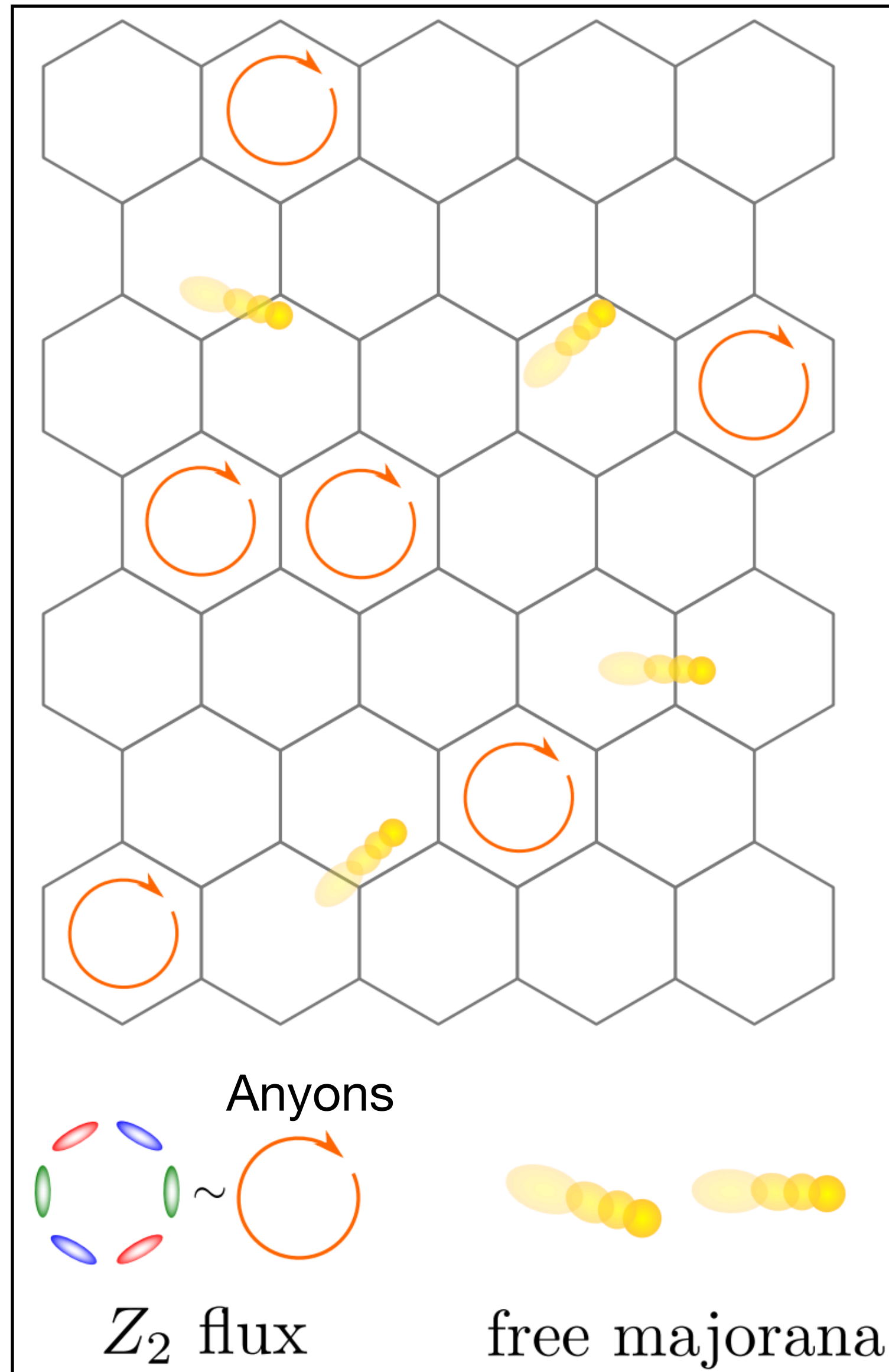
*S. Feng, A. Agarwala, S. Bhattacharjee, N. Trivedi. Phys. Rev. B 108, 035149 (2023)*

# Fascinating Phenomena in Quantum Materials: Fractionalization

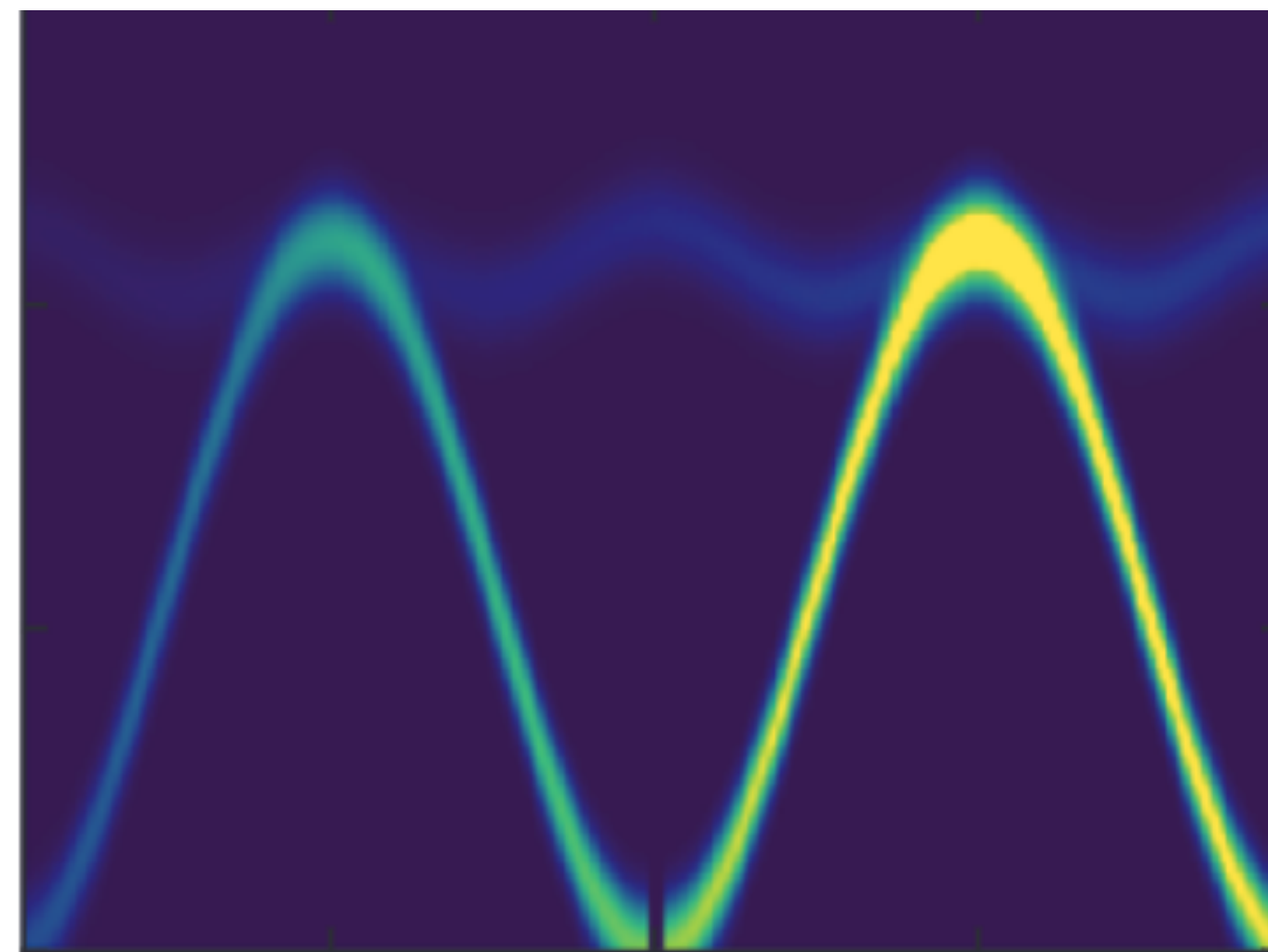


Can we detect any sharp features of fractionalized excitations?

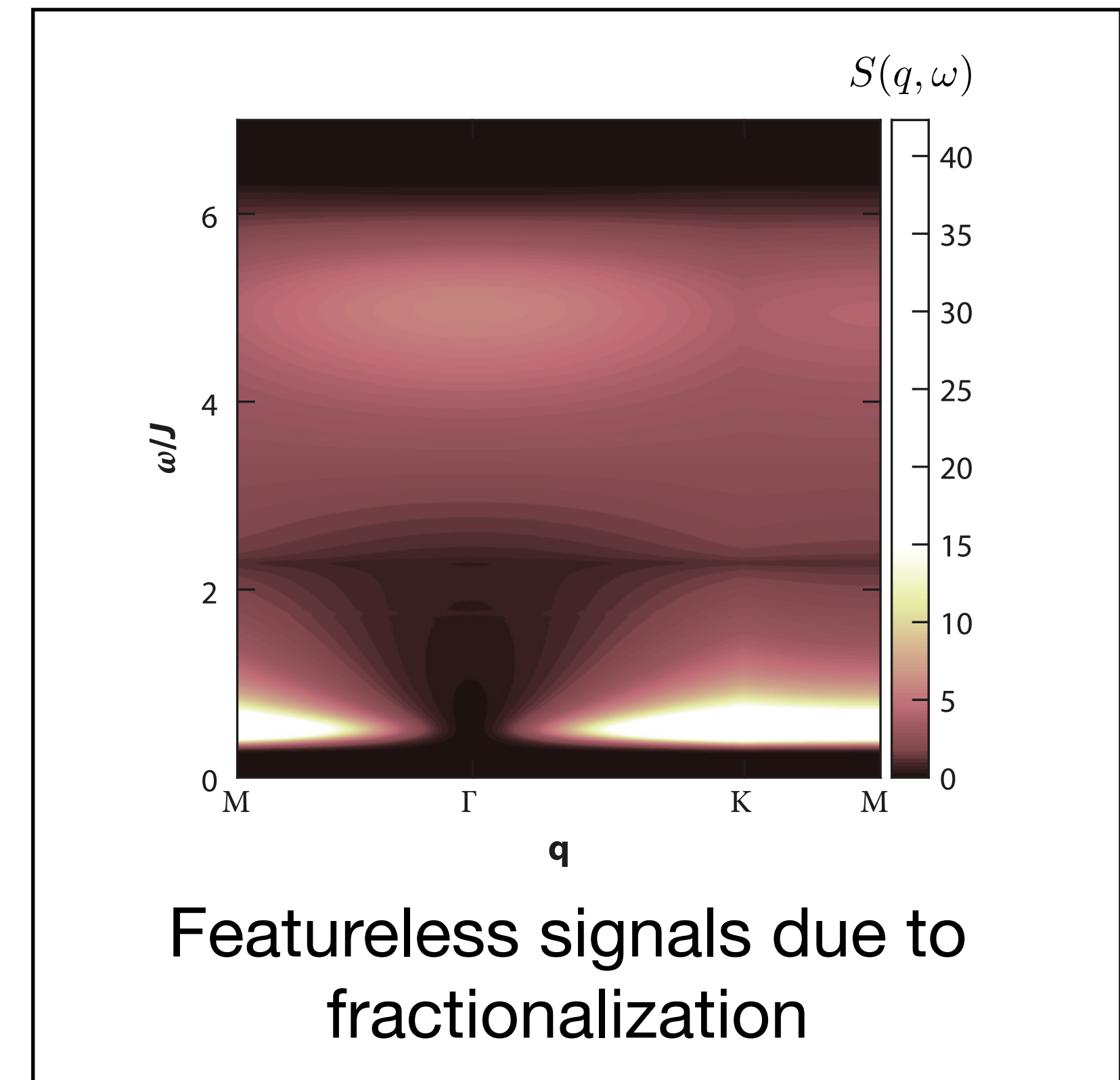
# Challenge in scattering experiments for QSL



Magnon spectra of CrSBr

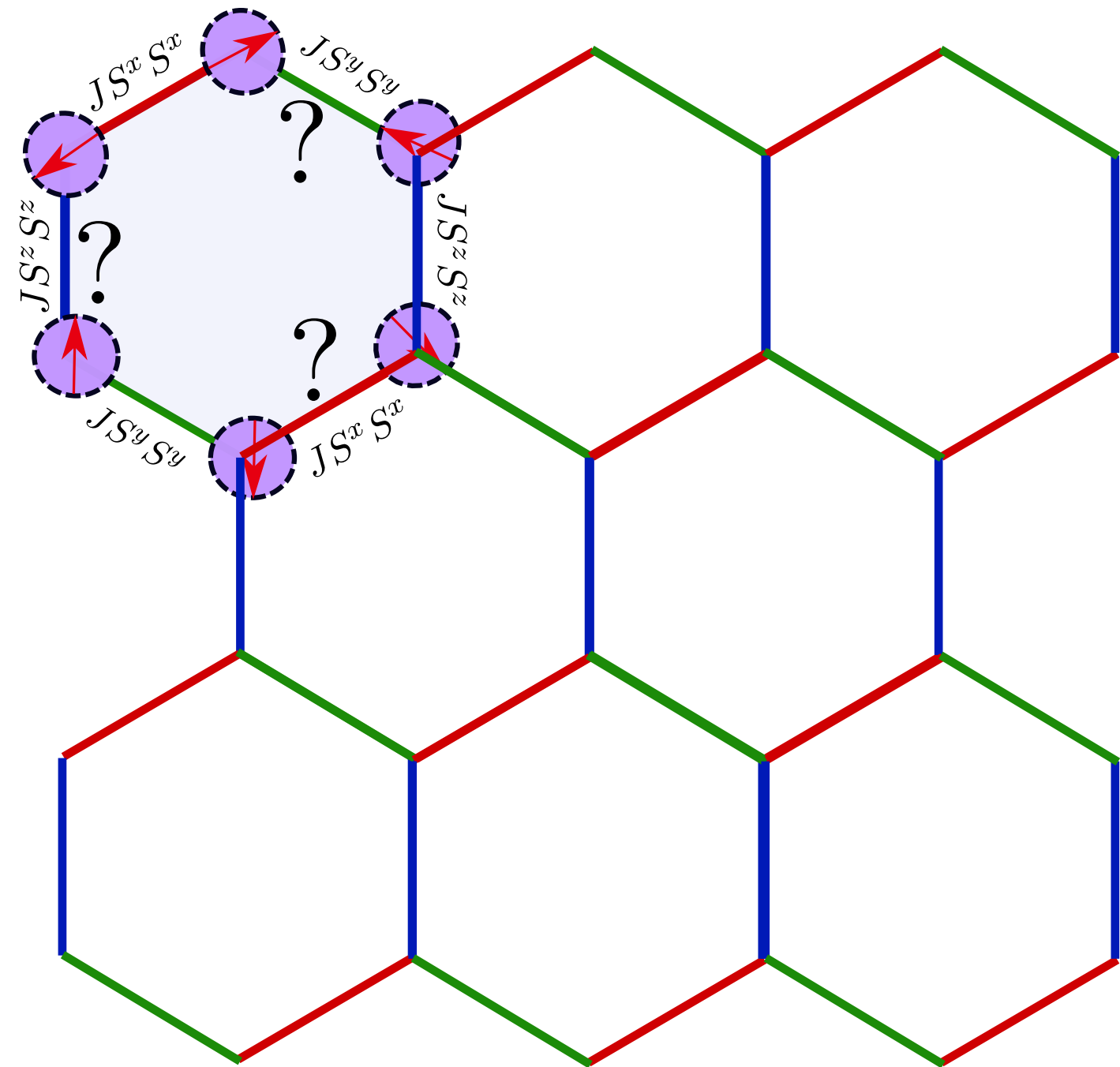


Sharp magnon modes without fractionalization



Featureless signals due to fractionalization

# 2D Frustrated Magnets -> Quantum spin liquid

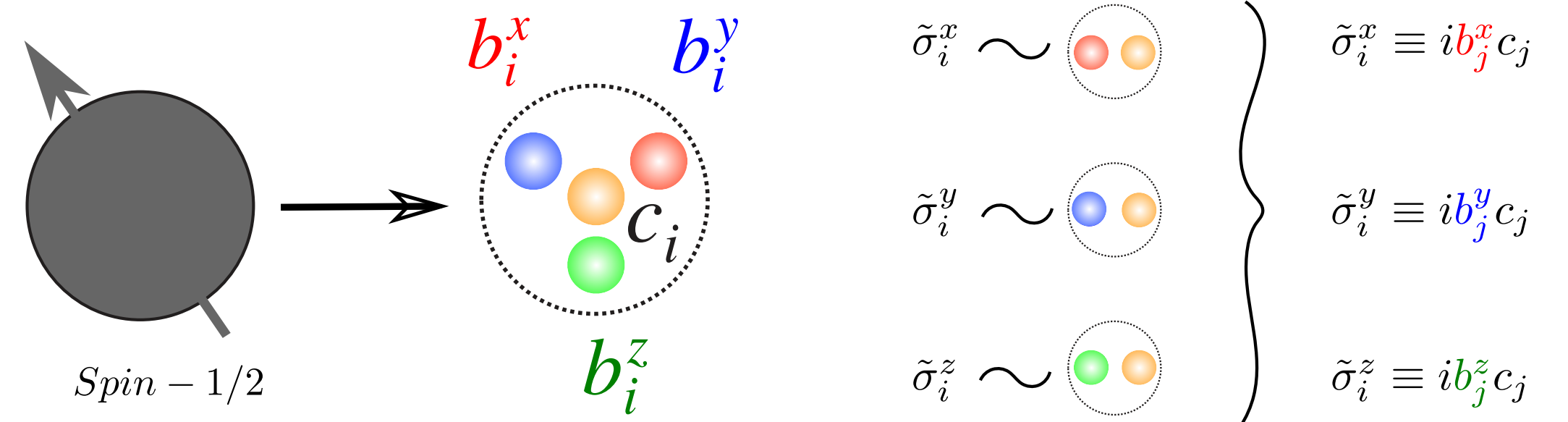


$$H = J \left( \sum_x S_i^x S_j^x + \sum_y S_i^y S_j^y + \sum_z S_i^z S_j^z \right)$$

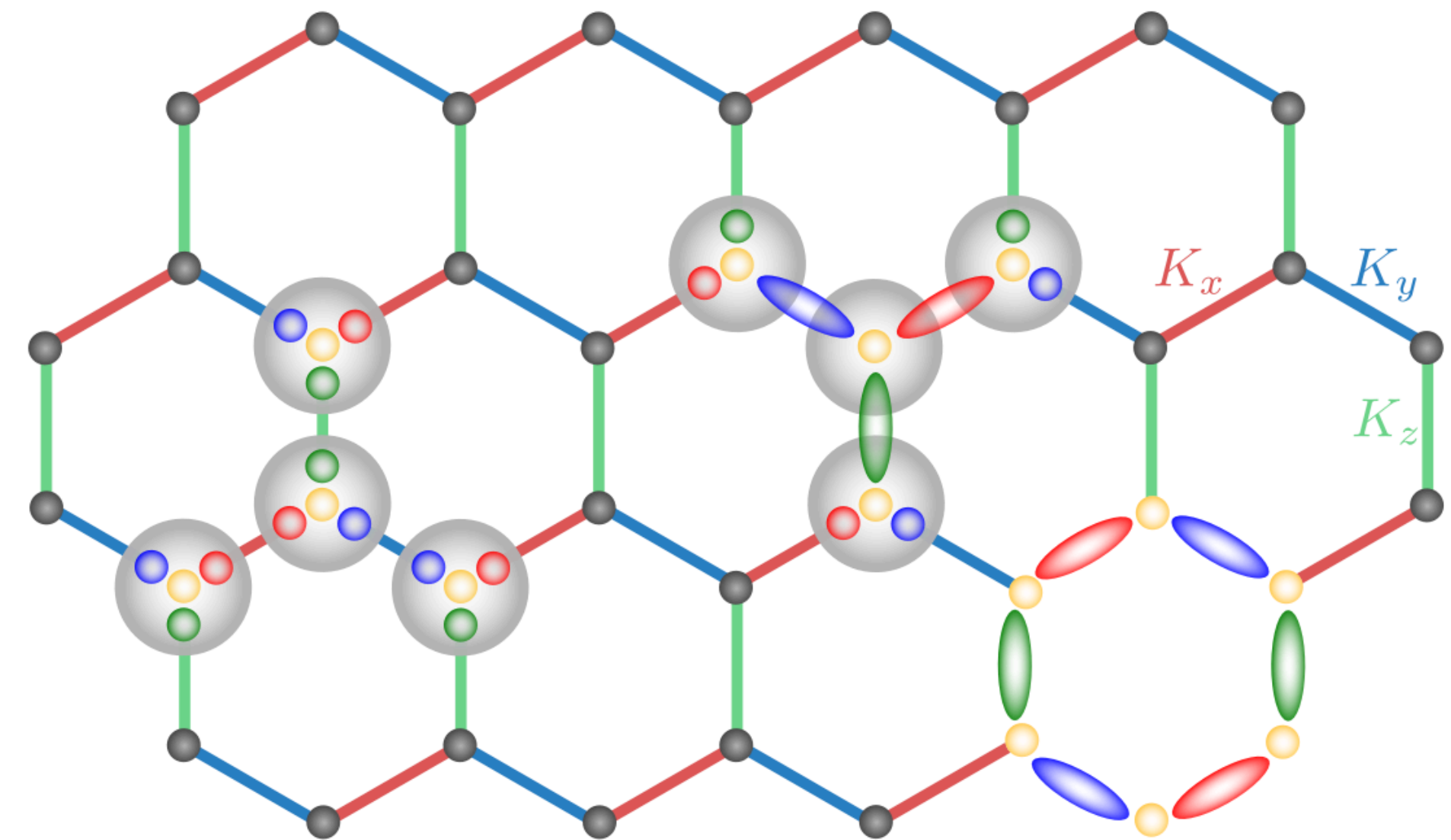
Kitaev's honeycomb model

## Dynamics of fractionalized quasi-particles

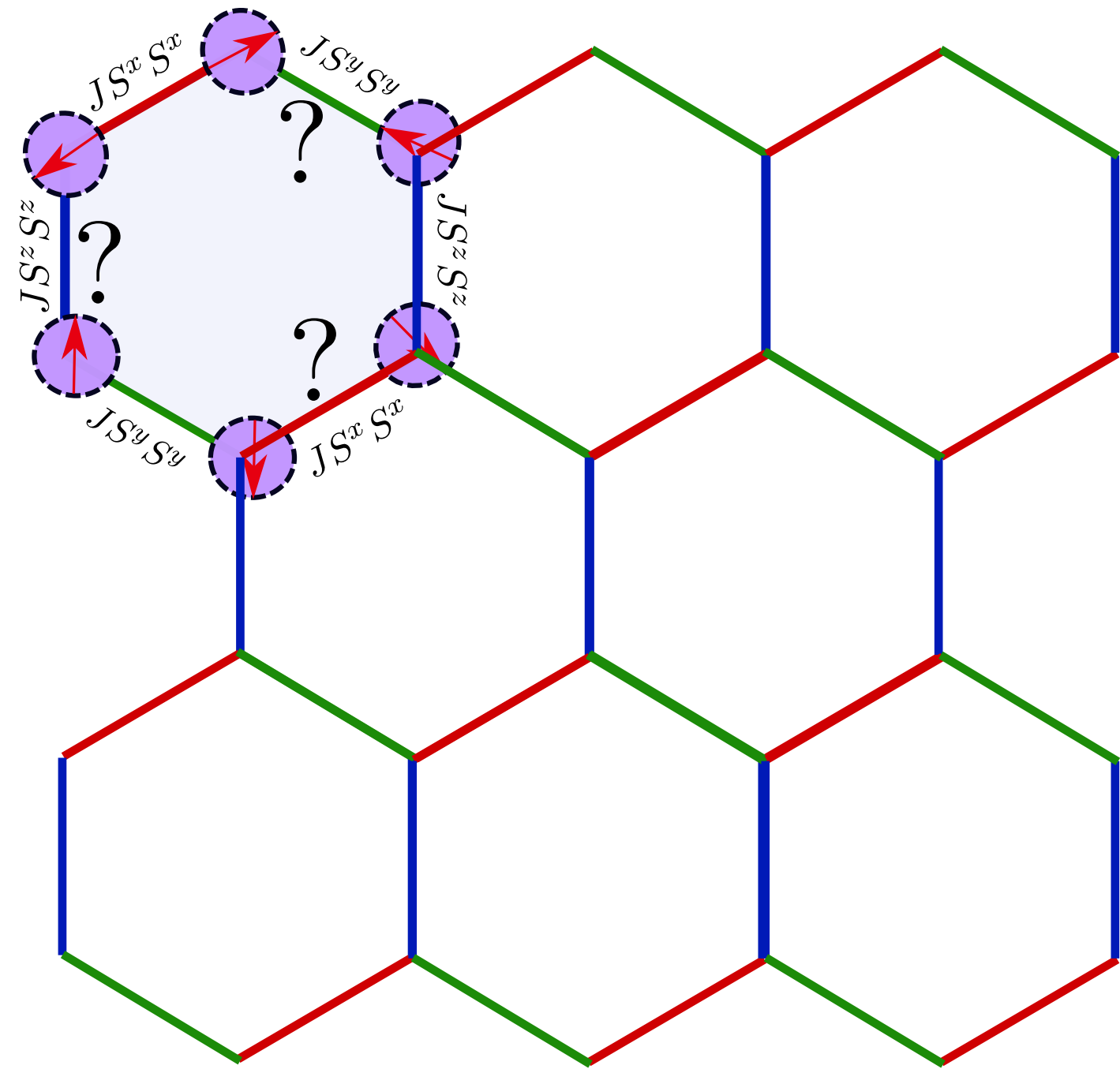
- Fractionalized particles



A. Kitaev, *Annals of Physics* 321 (2006) 2-111

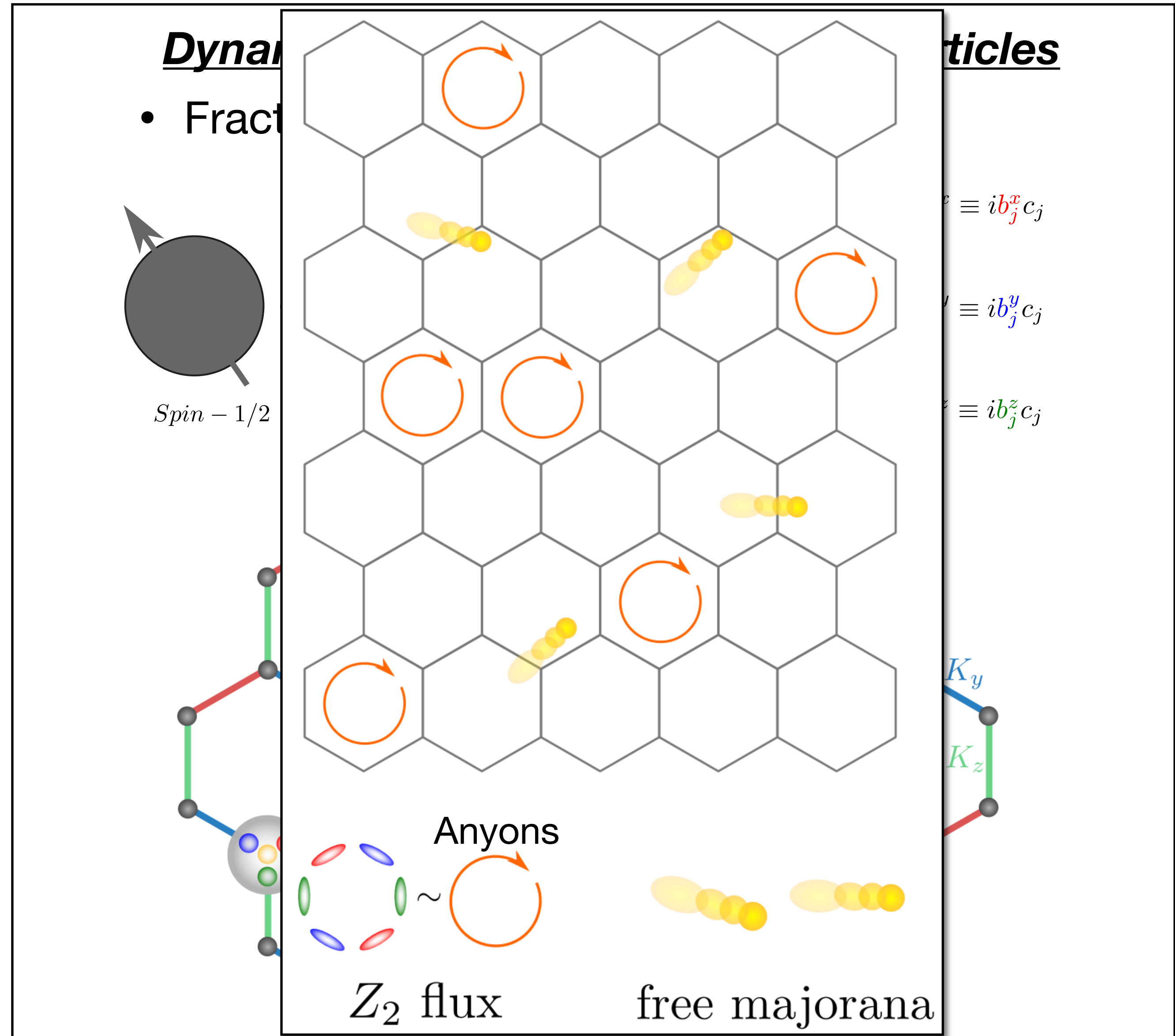


# 2D Frustrated Magnets -> Quantum spin liquid



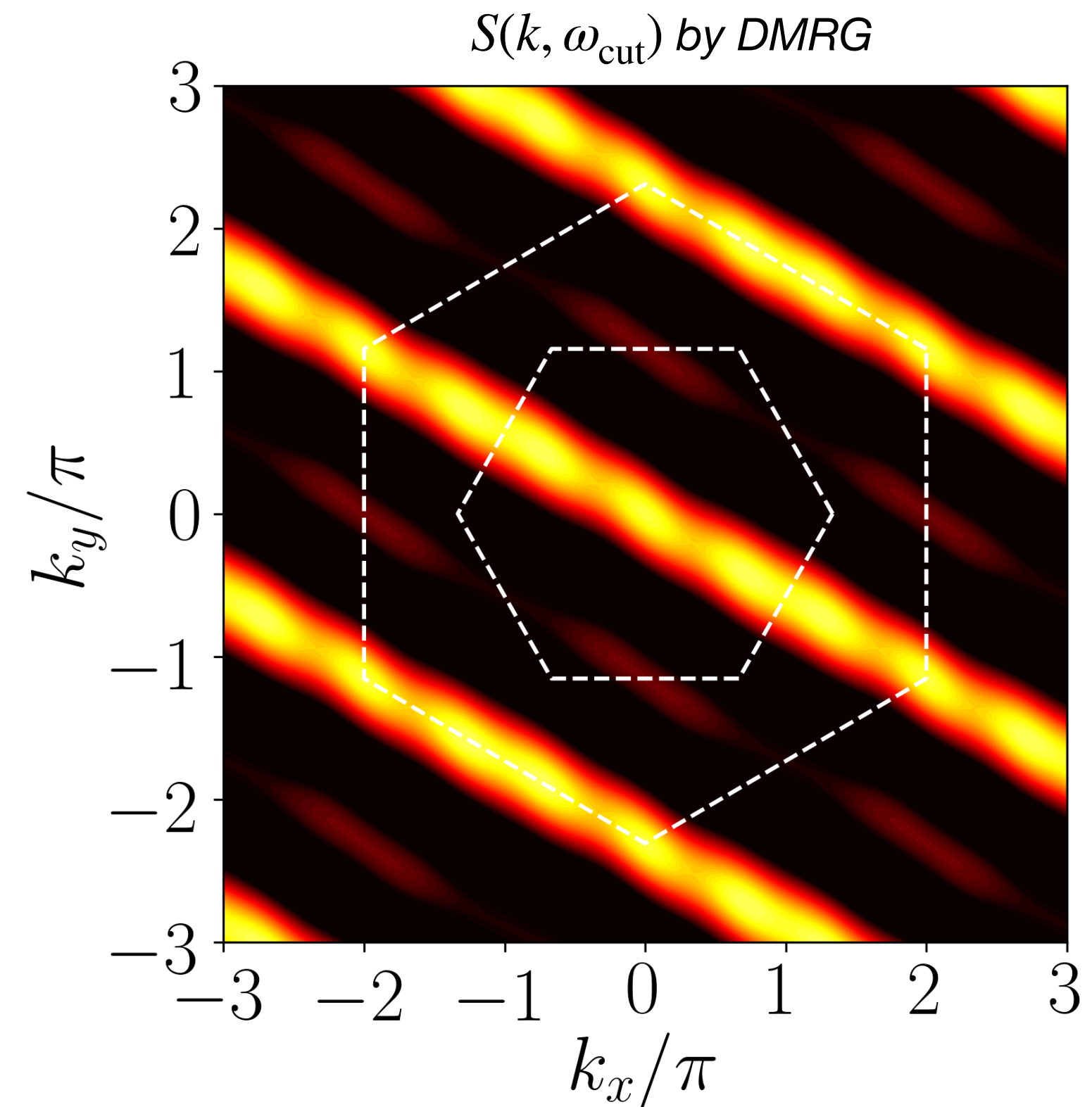
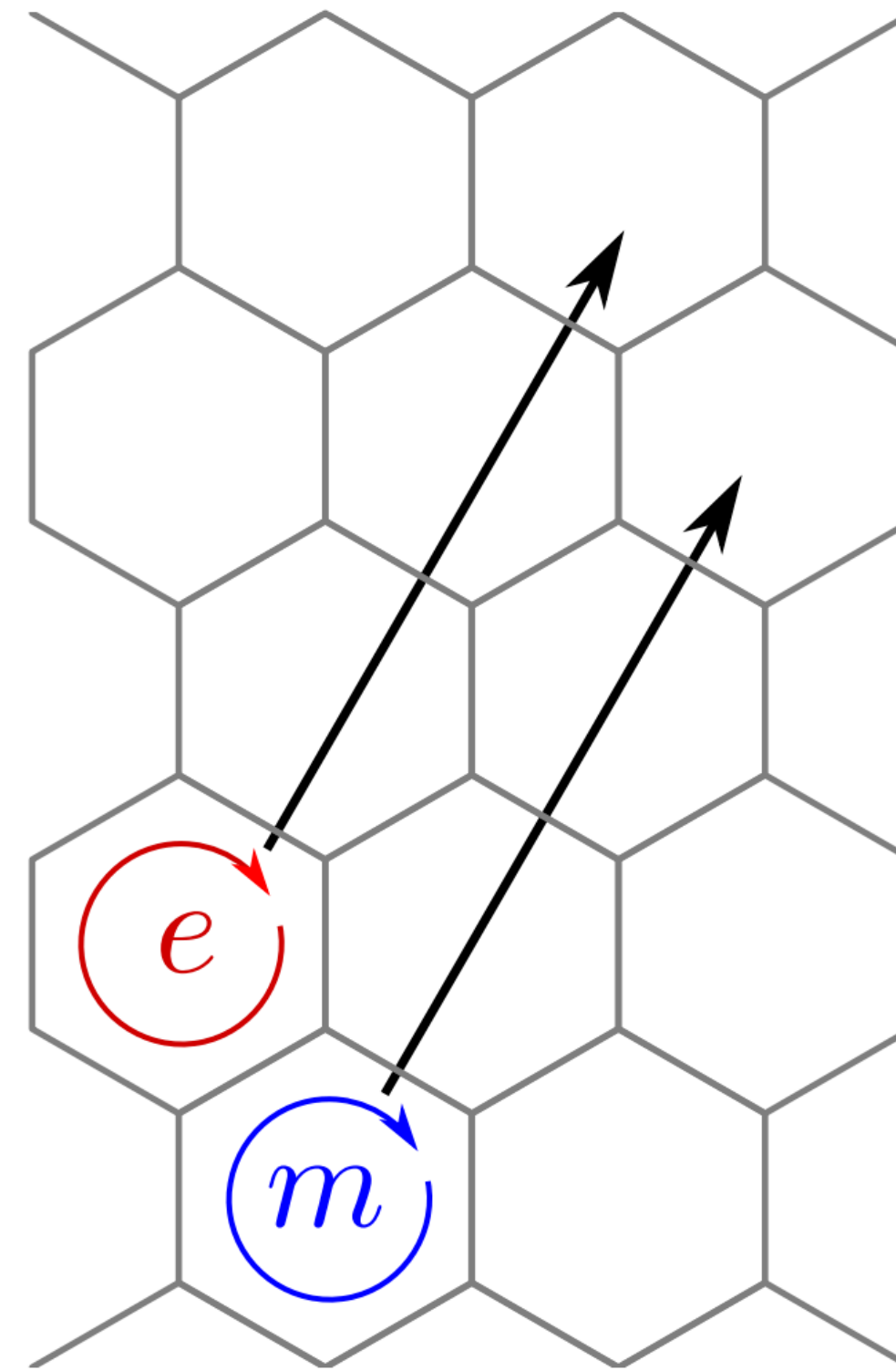
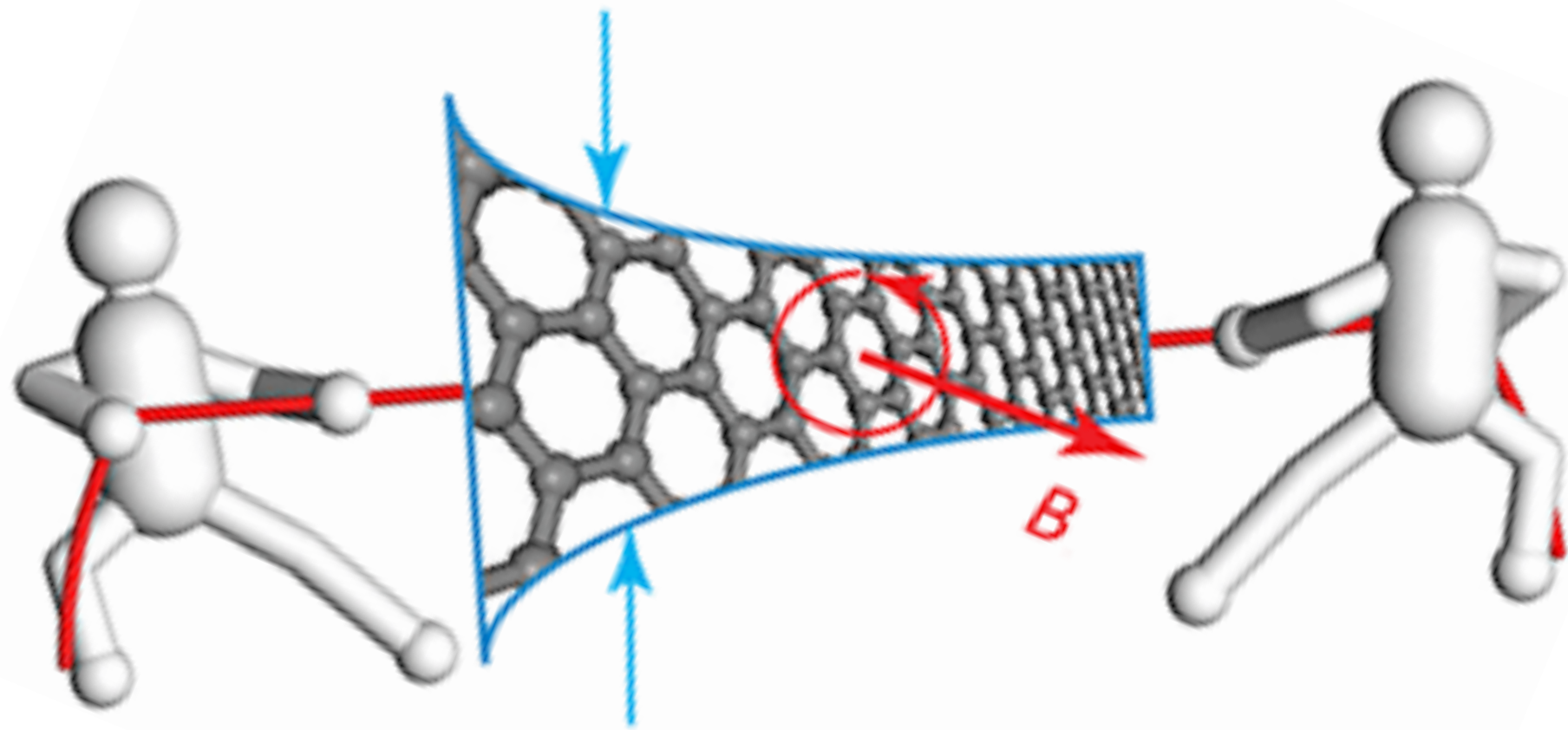
$$H = J \left( \sum_x S_i^x S_j^x + \sum_y S_i^y S_j^y + \sum_z S_i^z S_j^z \right)$$

Kitaev's honeycomb model



# Strain + Field: sharp signature of anyons

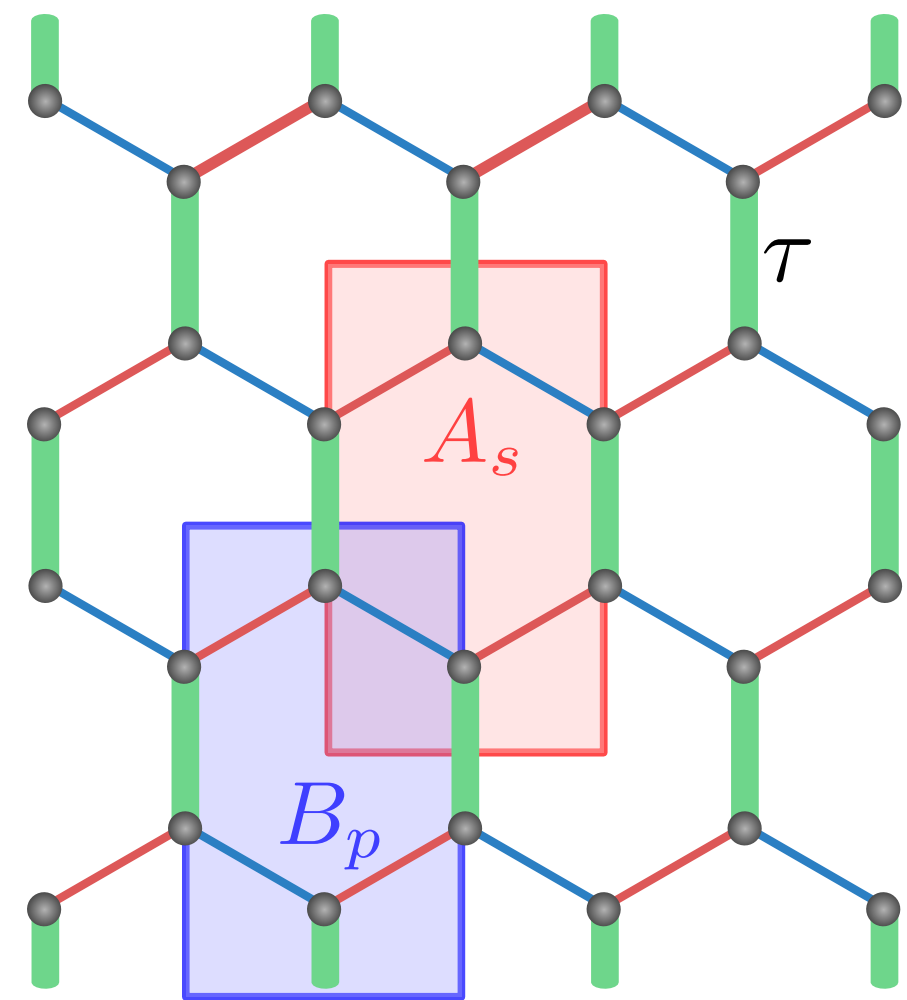
$$H = \sum_x K_x \sigma_i^x \sigma_{i+x}^x + \sum_y K_y \sigma_i^y \sigma_{i+y}^y + \sum_z K_z \sigma_i^z \sigma_{i+z}^z - B \sum_i \sigma_i^{e_3}$$



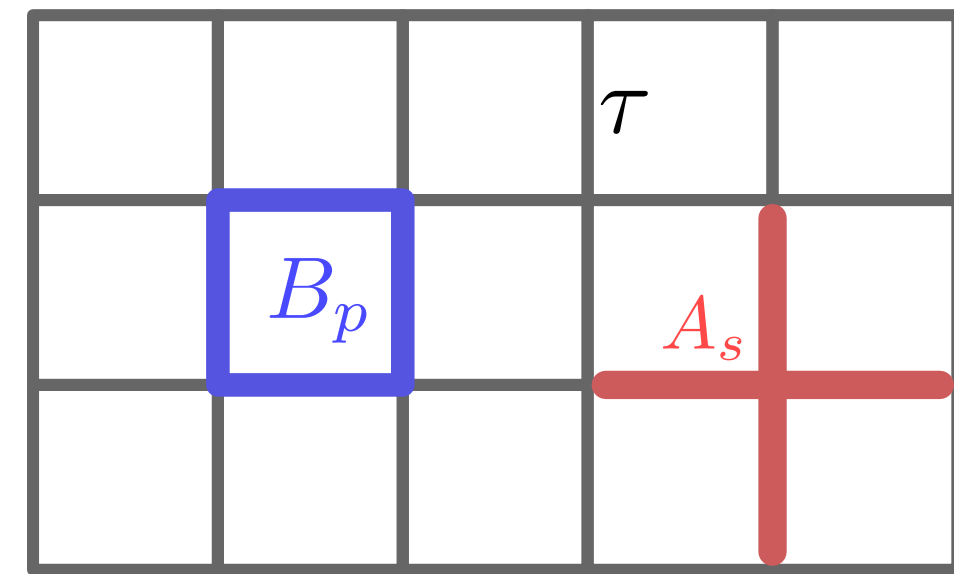
**1D fracton-like mobility of  $\epsilon = e \times m$  anyons bound state!**

# Effects of Strain $\rightarrow$ Toric Code + Majoranas

$$H_K = \sum_x K_x \sigma_i^x \sigma_{i+x}^x + \sum_y K_y \sigma_i^y \sigma_{i+y}^y + \sum_z K_z \sigma_i^z \sigma_{i+z}^z$$



$$K_z \gg K_x, K_y \quad \left[ \begin{array}{c} \uparrow \\ | \\ \downarrow \end{array} \right] \in \{|\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle\} \sim \tau$$



$$\mathcal{H}_{\text{TC}} \sim - \sum_s A_s - \sum_p B_p$$

Low energy sector:  
Toric Code (TC)  $A_s = \prod_{i \in +} \tau_i^x, B_p = \prod_{i \in \square} \tau_i^z, [A_s, B_p] = 0$

ground state :  $A_s = B_p = 1$

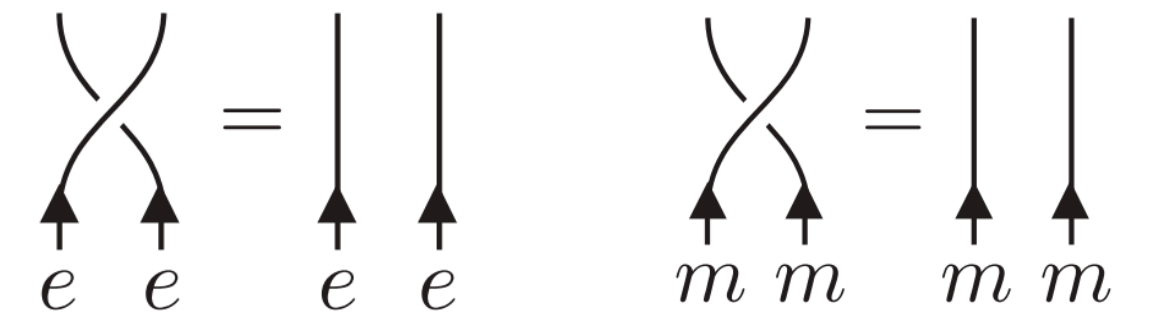
Fractionalization:

1.  $Z_2$  Abelian anyons:

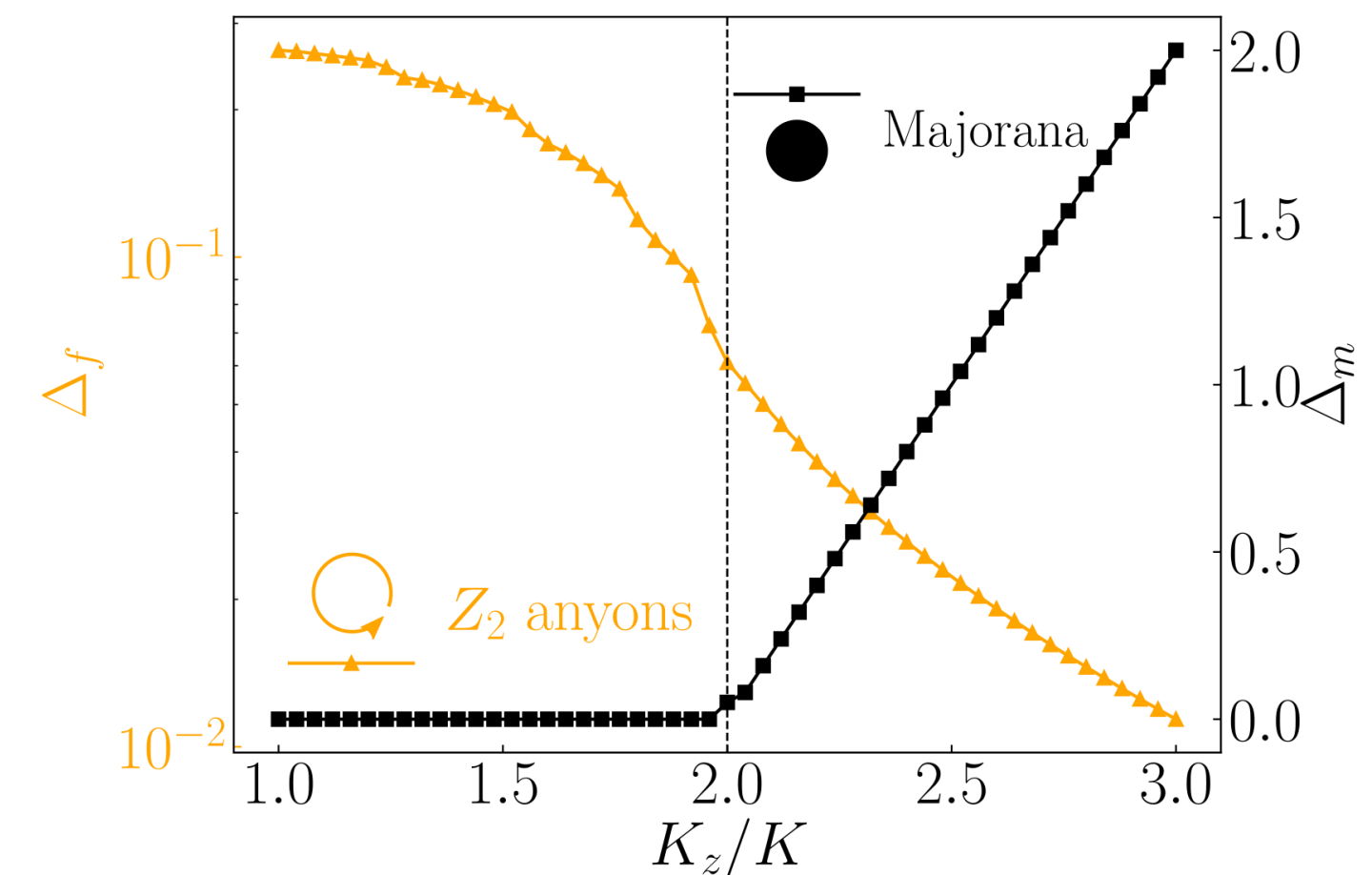
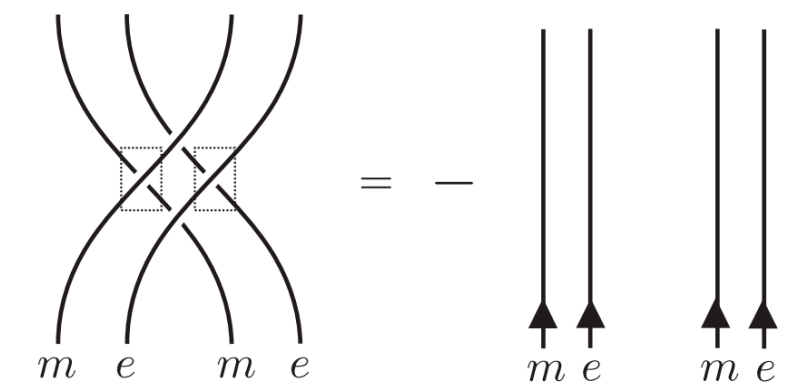
•  $e$  ( $A_s = -1$ ) boson

•  $m$  ( $B_p = -1$ ) boson

•  $\epsilon = e \times m$  fermion



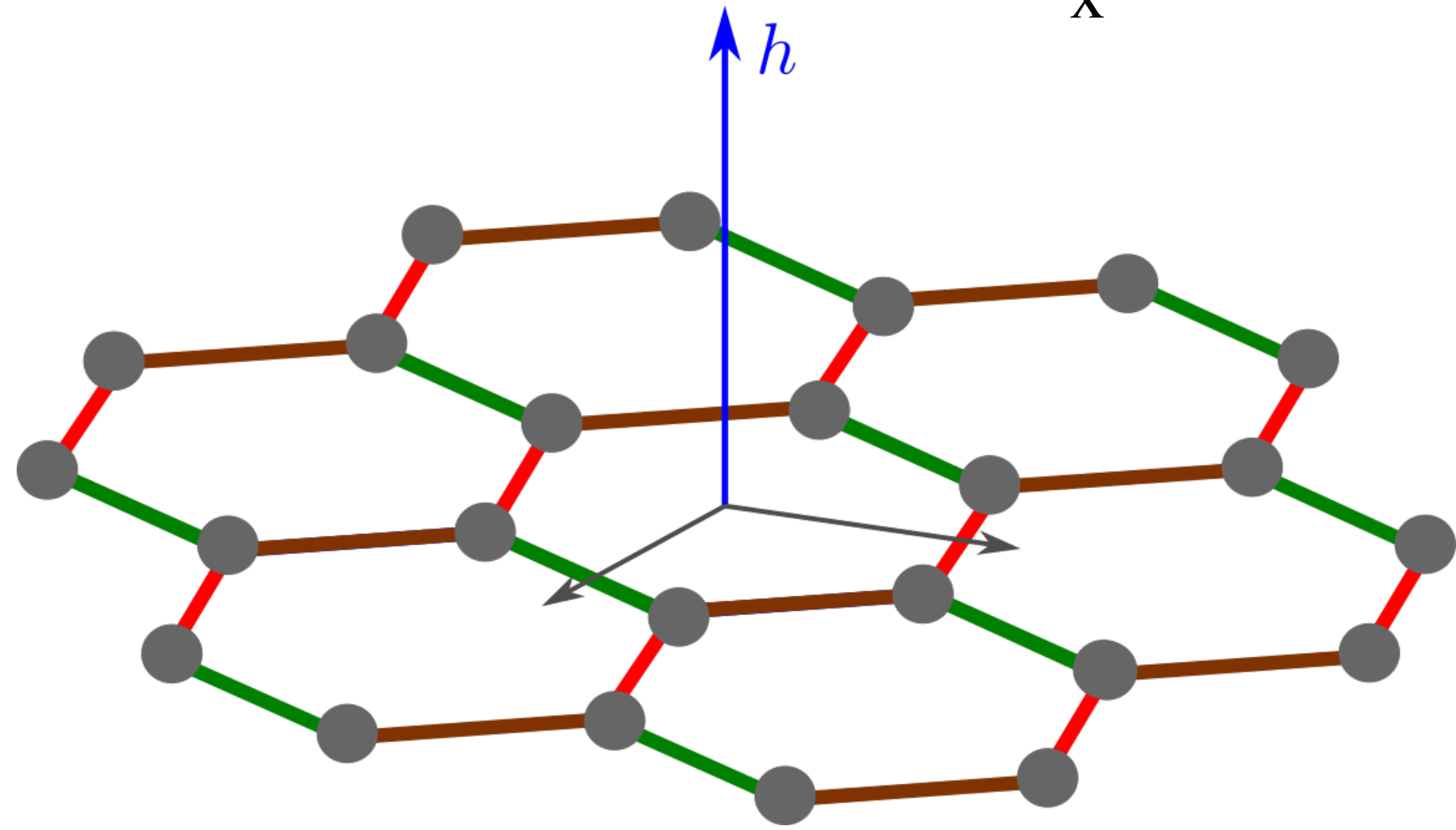
2. Gapped Majorana fermions



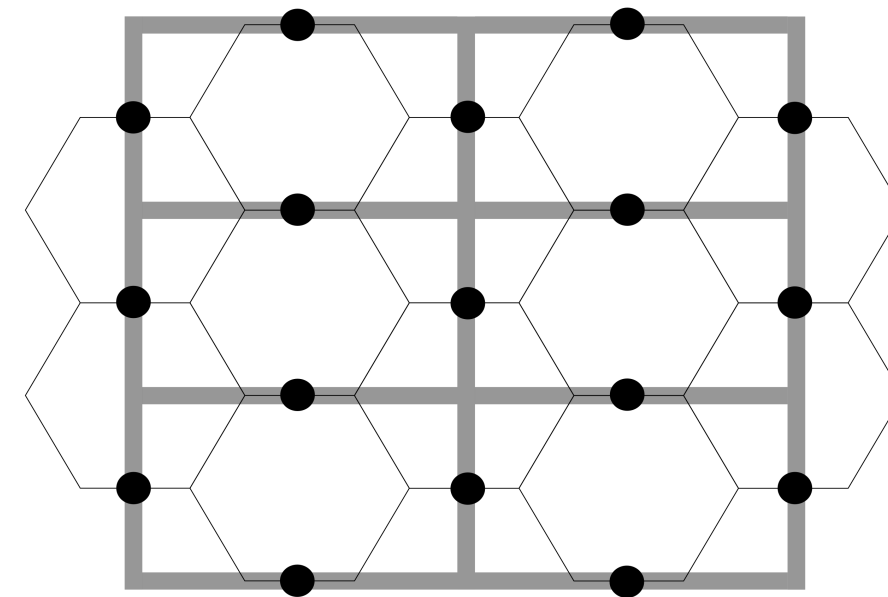
Low energy physics is dominated by **anyons**

# Magnetic-field-induced anyon dynamics

$$H = \sum_x K_x \sigma_i^x \sigma_{i+x}^x + \sum_y K_y \sigma_i^y \sigma_{i+y}^y + \sum_z K_z \sigma_i^z \sigma_{i+z}^z - h \sum_i \sigma_i^{e_3}$$



Second order perturbation theory:



A vertical line with a central black dot and four arrows pointing outwards (up, down, left, right). This is defined as:

$$\text{Diagram} = |\uparrow\downarrow\rangle \text{ or } |\downarrow\uparrow\rangle \longrightarrow \tau$$

1 spin flip  $\rightarrow$  high energy sector  
 2 spin flips  $\rightarrow$  returns to the low energy sector

$$H_{\text{eff}} \sim - \sum_s A_s - \sum_p B_p + \sum_i \frac{h^2}{K_z} \tau_i^y$$



# Magnetic-field-induced anyon dynamics

$$H_{\text{eff}} \sim - \sum_s A_s - \sum_p B_p + \sum_i \frac{\hbar^2}{K_z} \tau_i^y$$

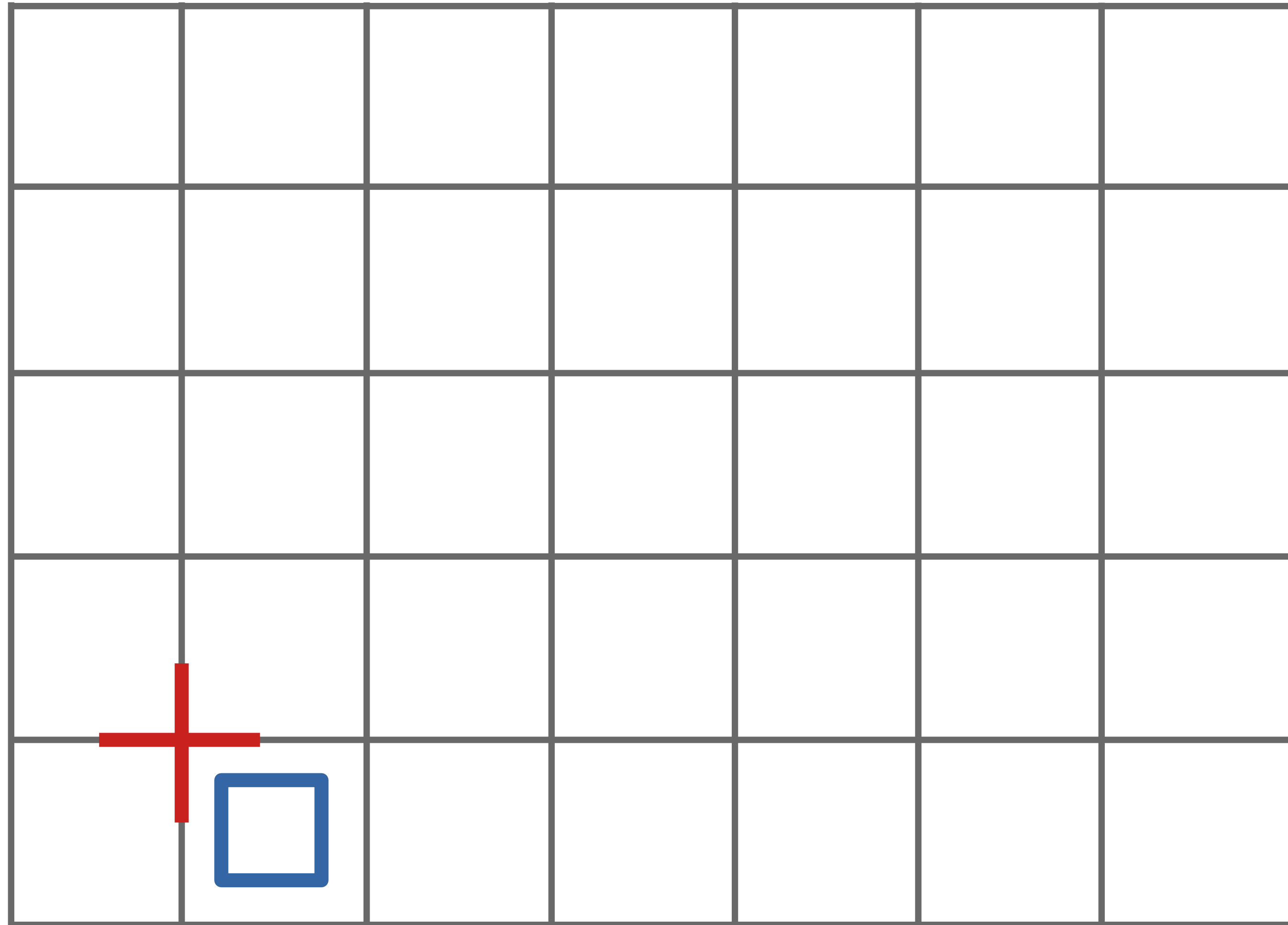
$$A_s = \prod_{i \in s} \tau_i^x$$

$$B_p = \prod_{i \in p} \tau_i^z$$

$$\tau_i^y \sim \tau_i^z \tau_i^x$$

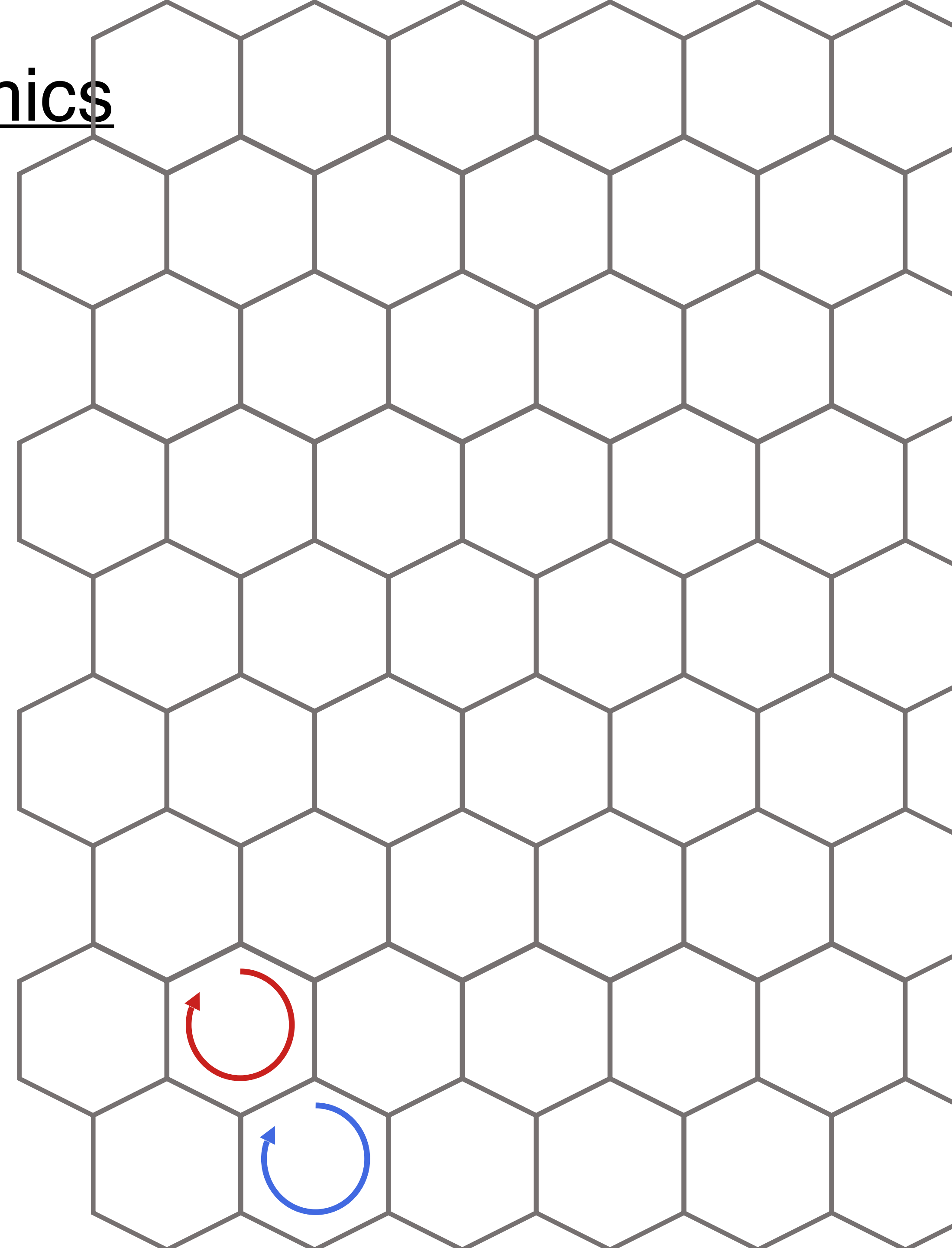
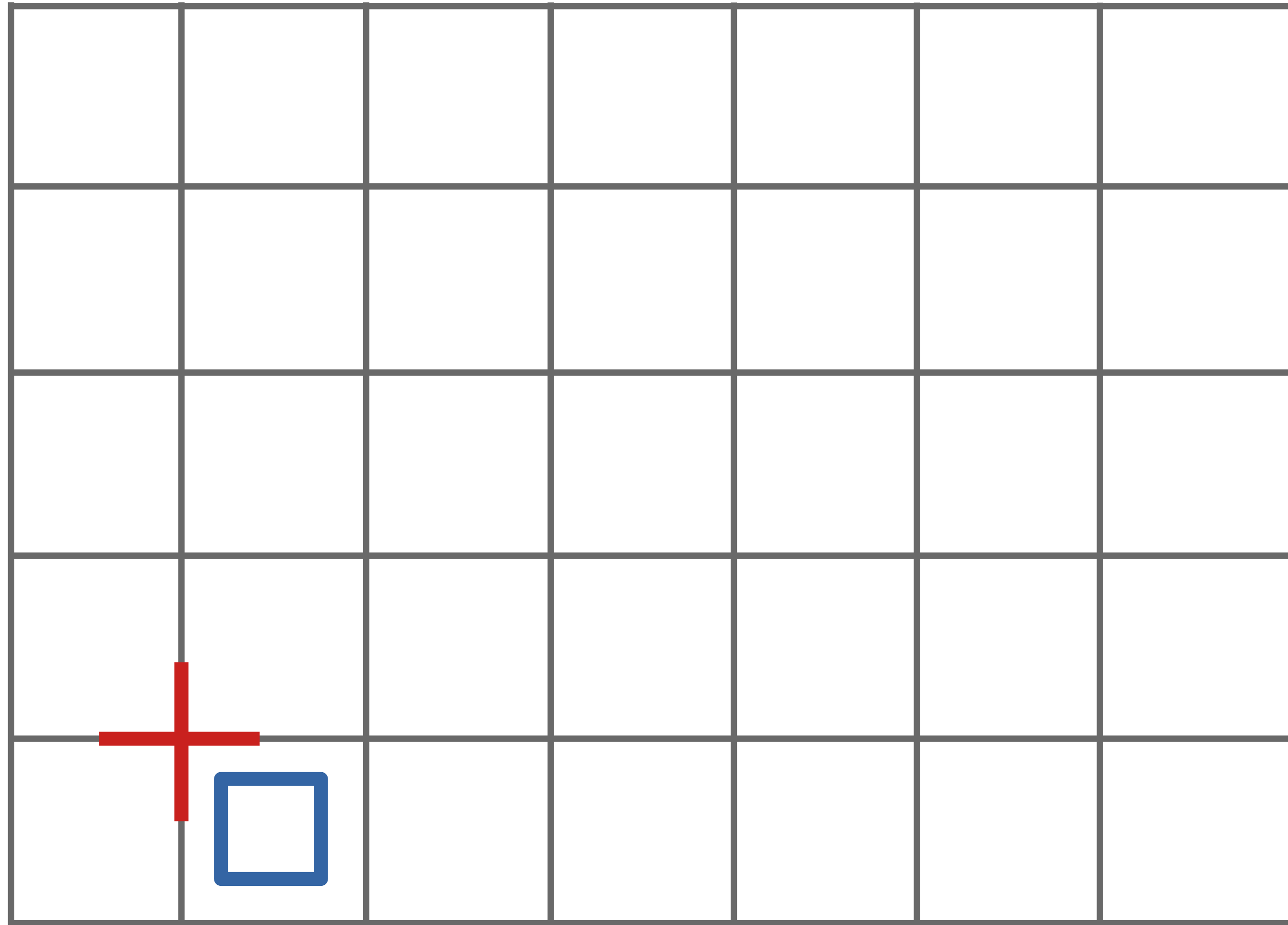
$$[\tau_i^x, A_s] = 0, \quad [\tau_i^x, B_p] \neq 0$$

$$[\tau_i^z, B_p] = 0, \quad [\tau_i^z, A_s] \neq 0$$



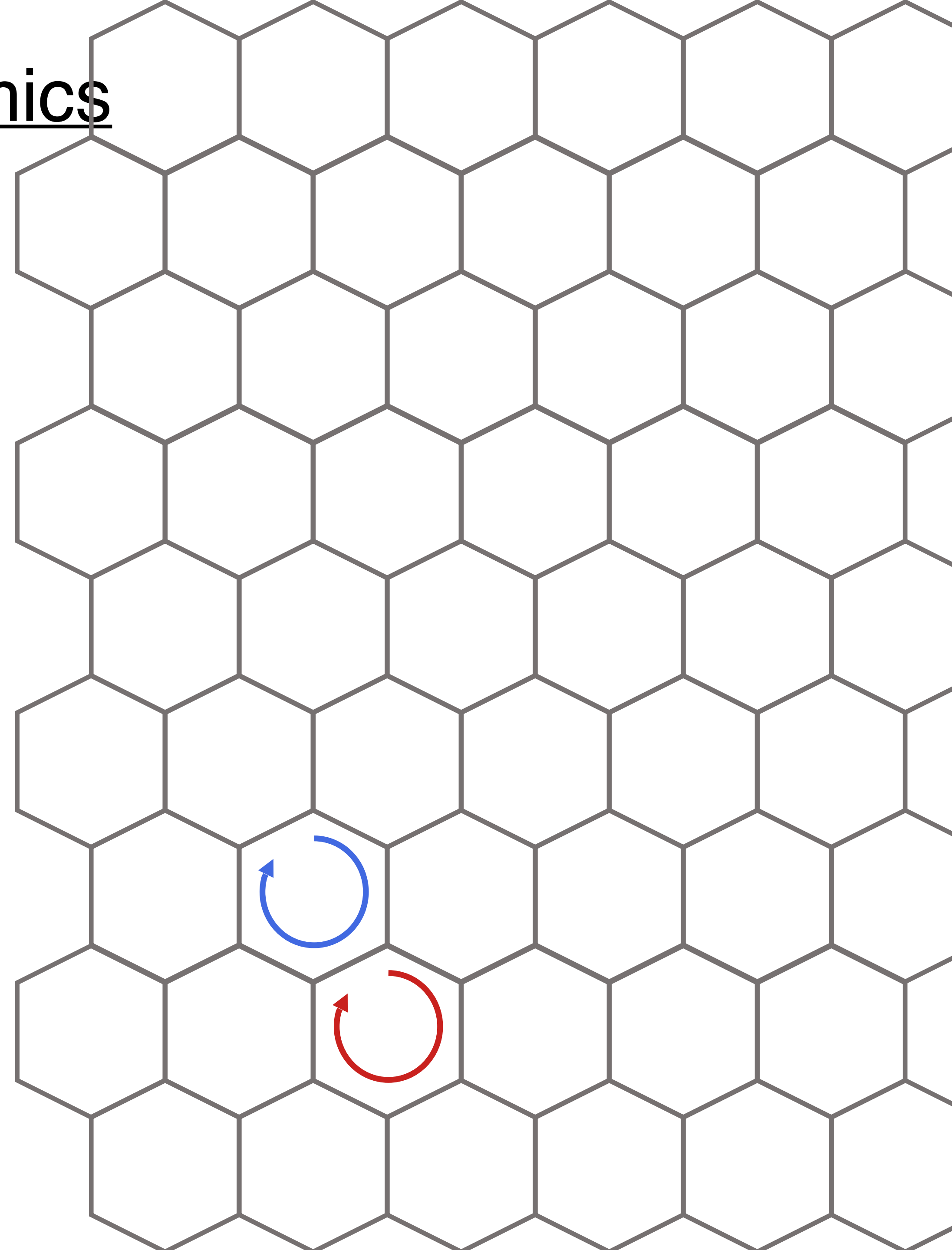
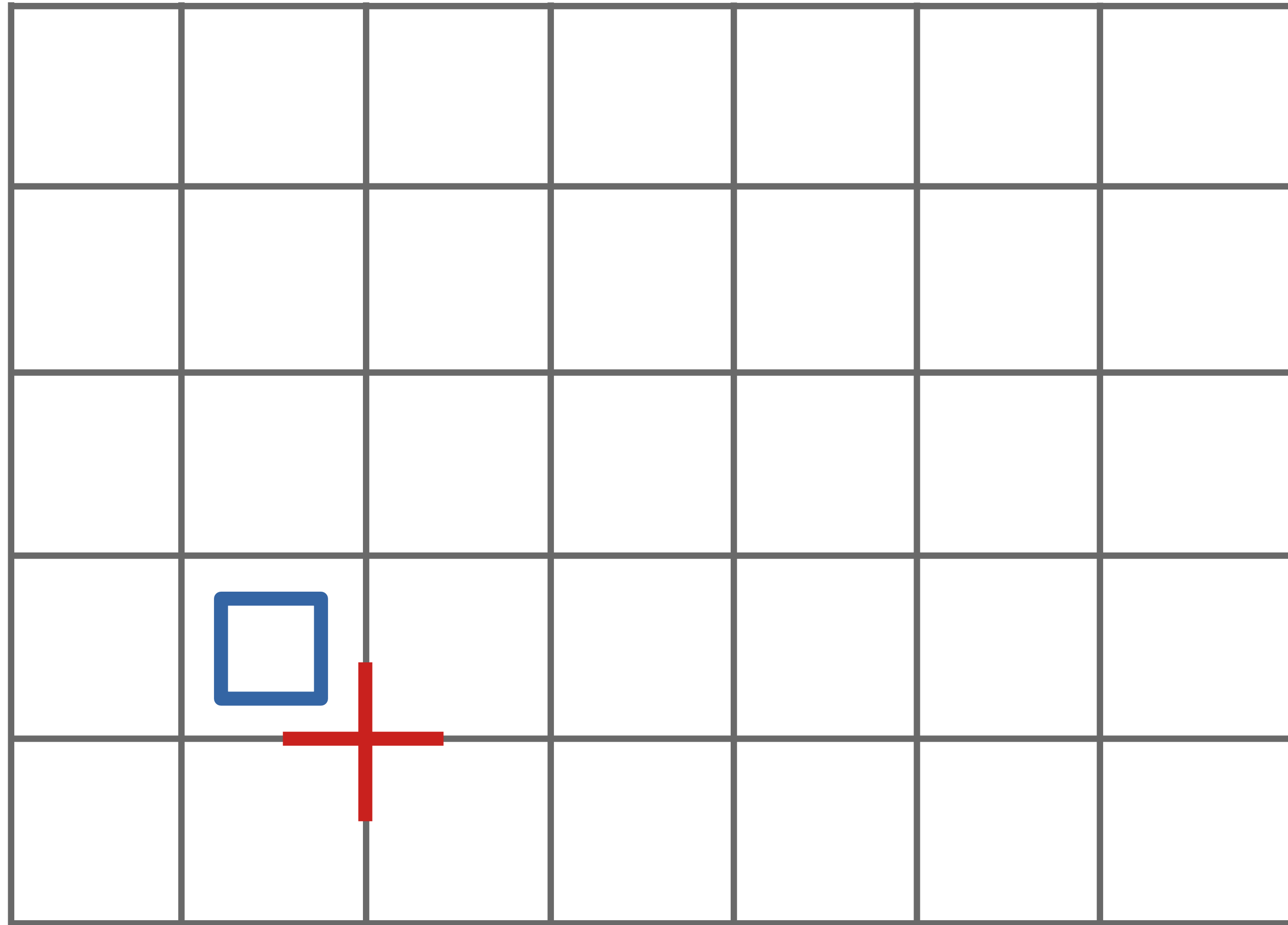
# Magnetic-field-induced anyon dynamics

$$H_{\text{eff}} \sim - \sum_s A_s - \sum_p B_p + \sum_i \frac{\hbar^2}{K_z} \tau_i^y$$



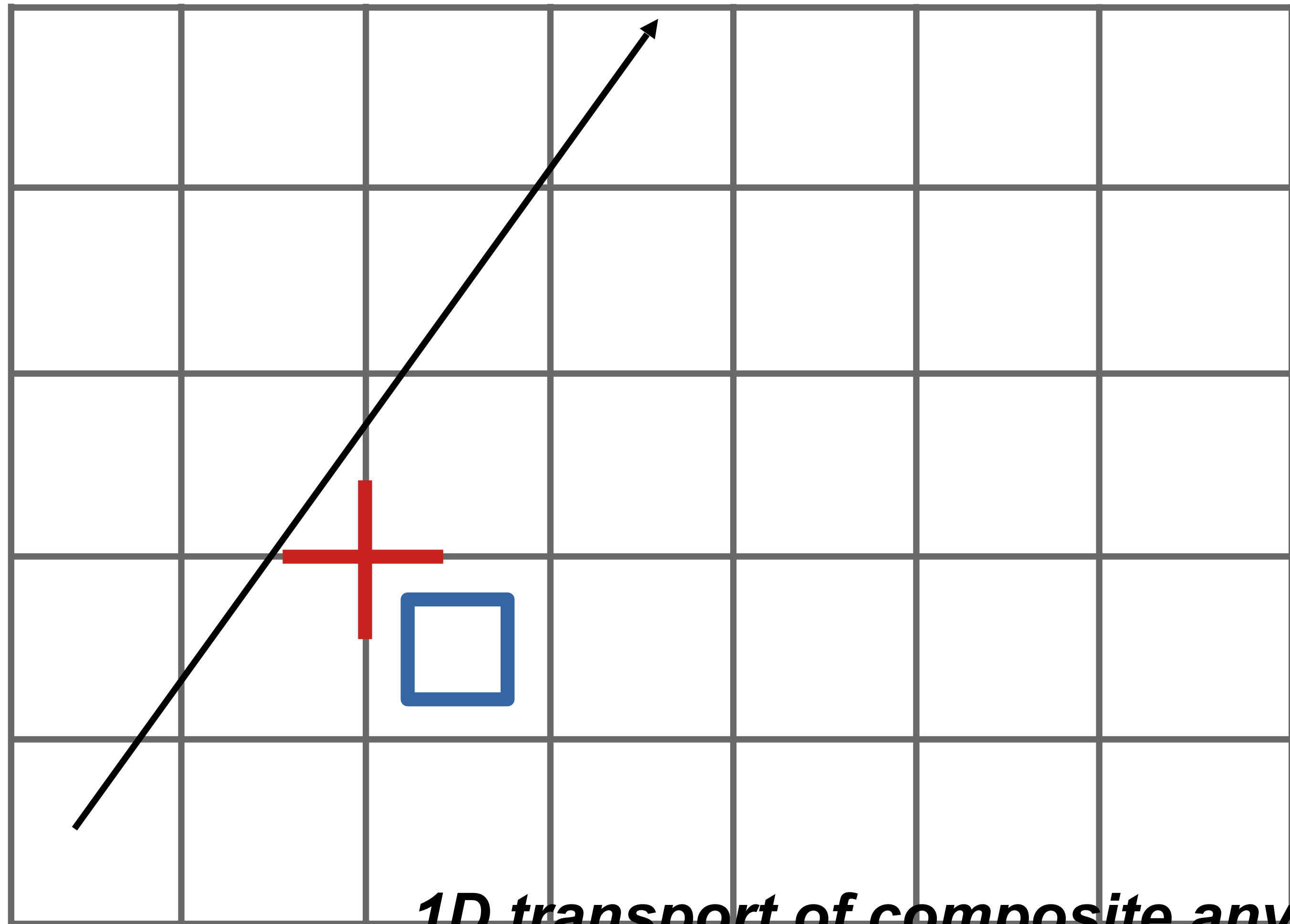
# Magnetic-field-induced anyon dynamics

$$H_{\text{eff}} \sim - \sum_s A_s - \sum_p B_p + \sum_i \frac{\hbar^2}{K_z} \tau_i^y$$

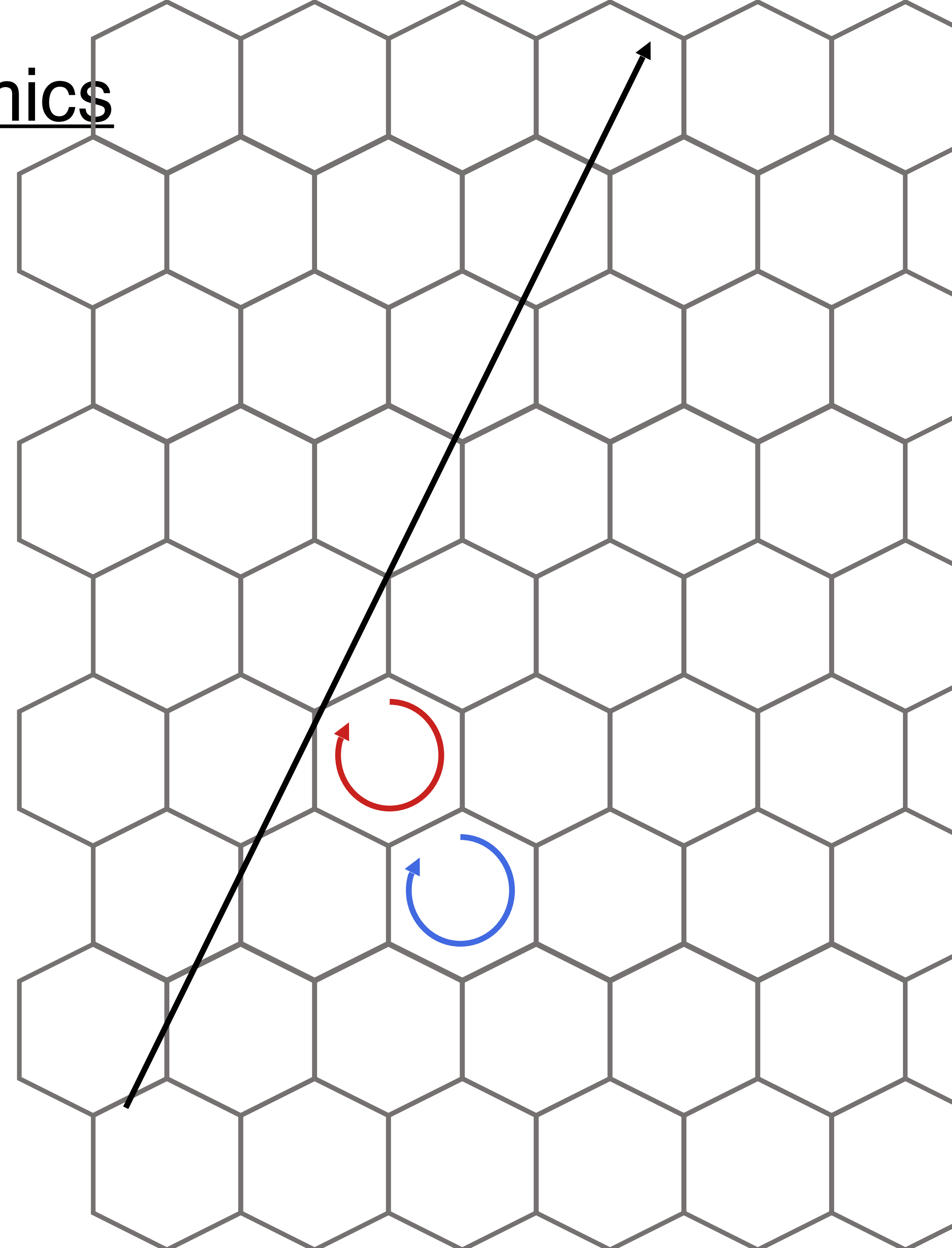


# Magnetic-field-induced anyon dynamics

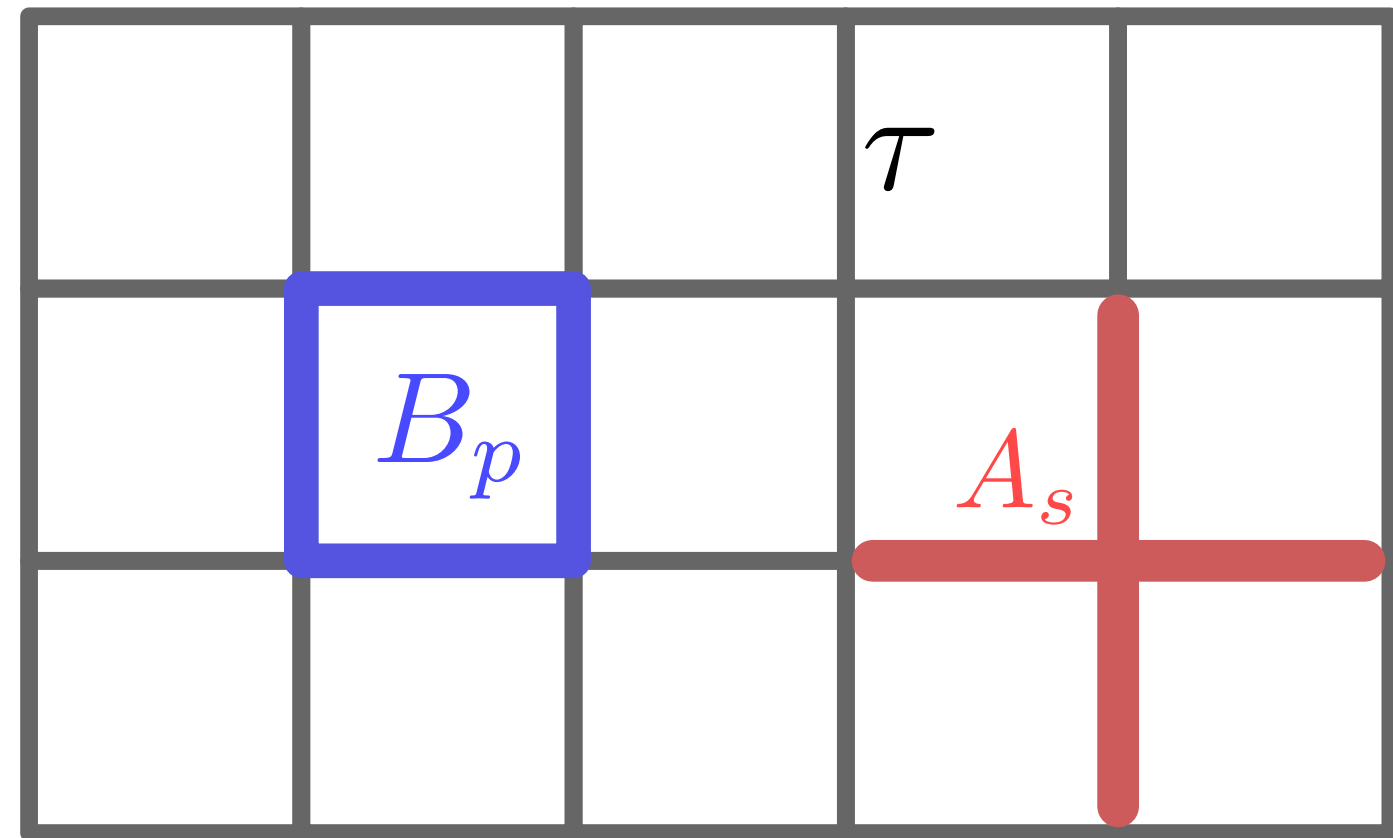
$$H_{\text{eff}} \sim - \sum_s A_s - \sum_p B_p + \sum_i \frac{\hbar^2}{K_z} \tau_i^y$$



***1D transport of composite anyon  
in a 2D model!***

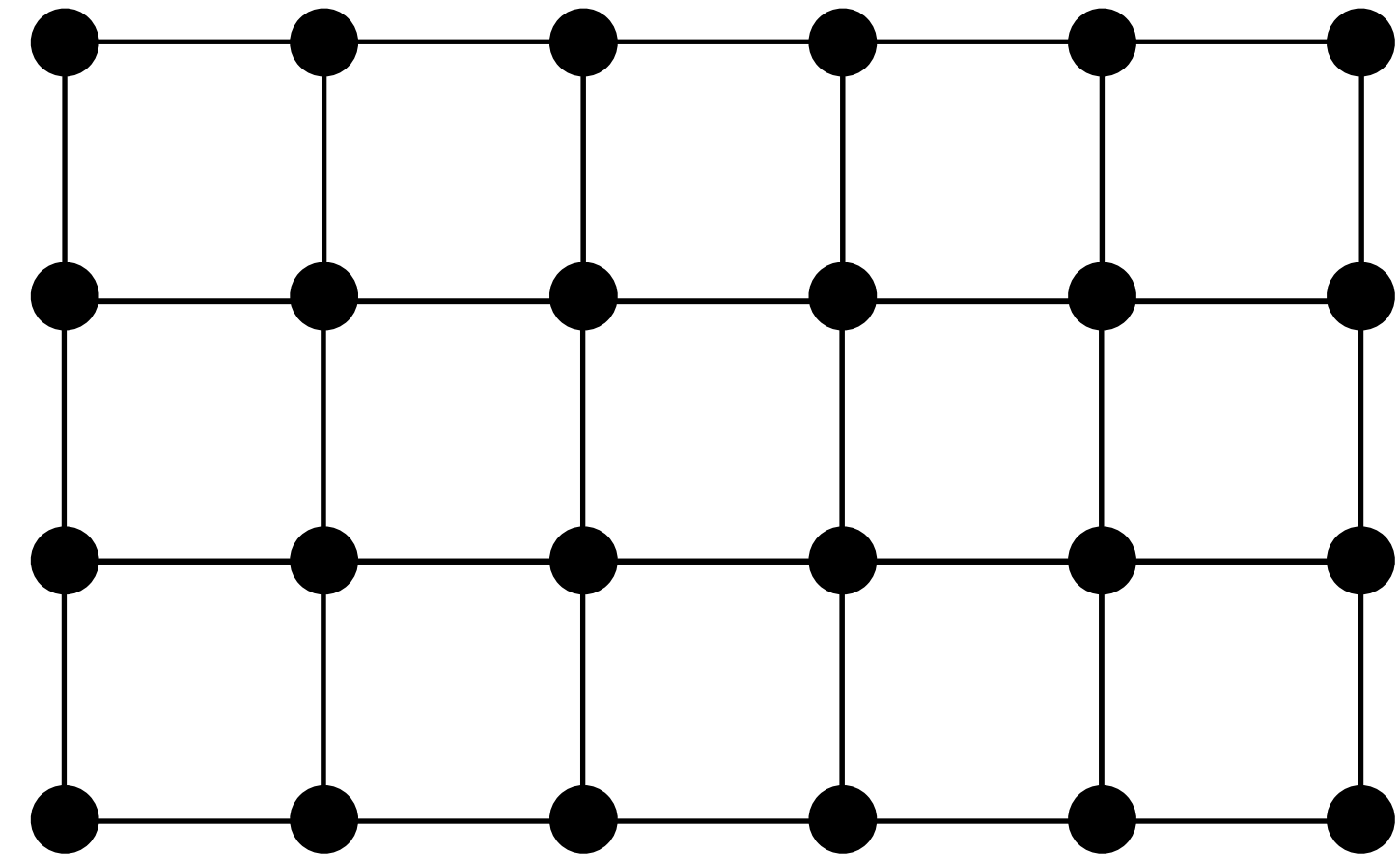


# Fractonic Physics by Duality

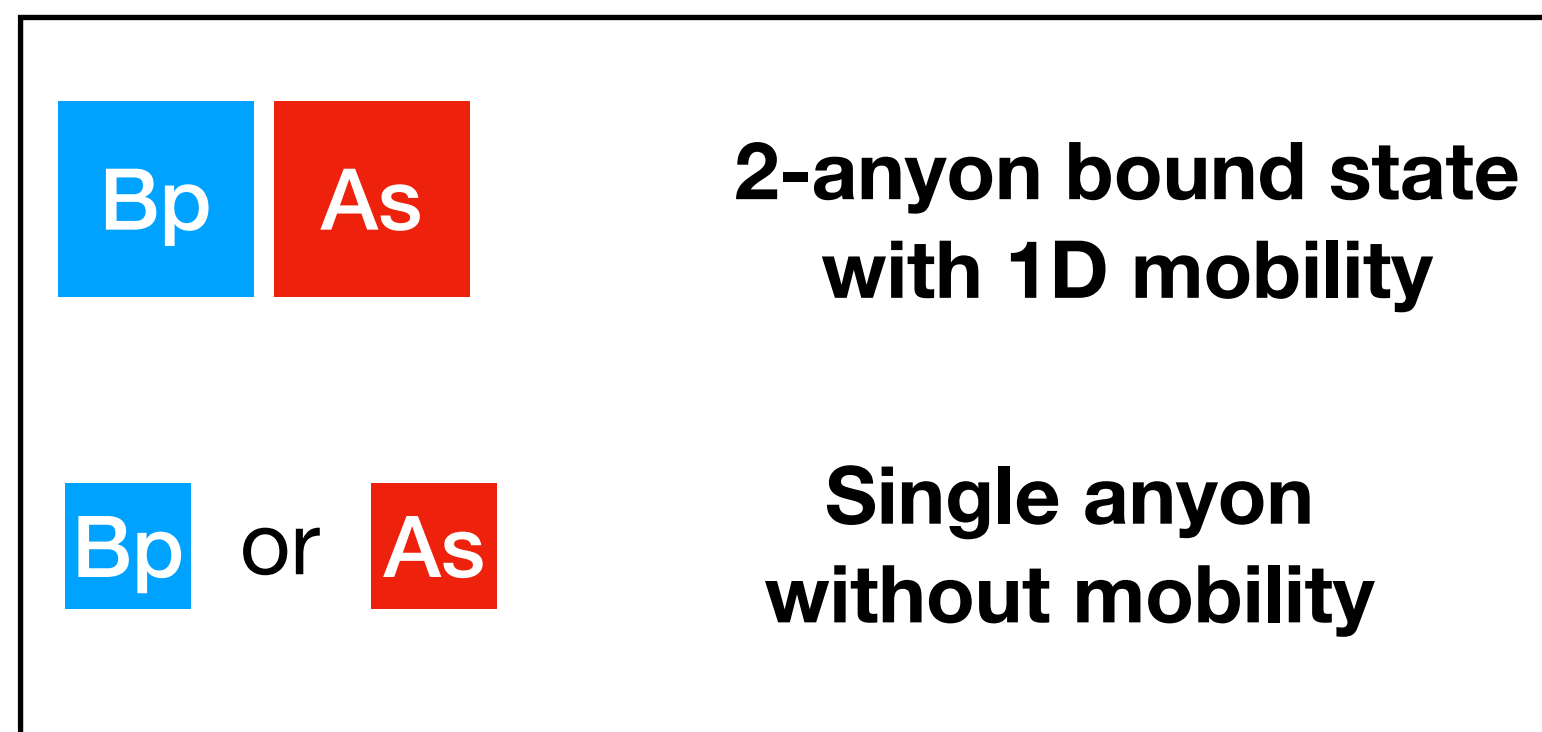


$$H_{K_z \gg K_{x,y}}^{\text{eff}} = H_{\text{TC}} - \frac{2h^2}{K_z} \sum_i \tau_i^y$$

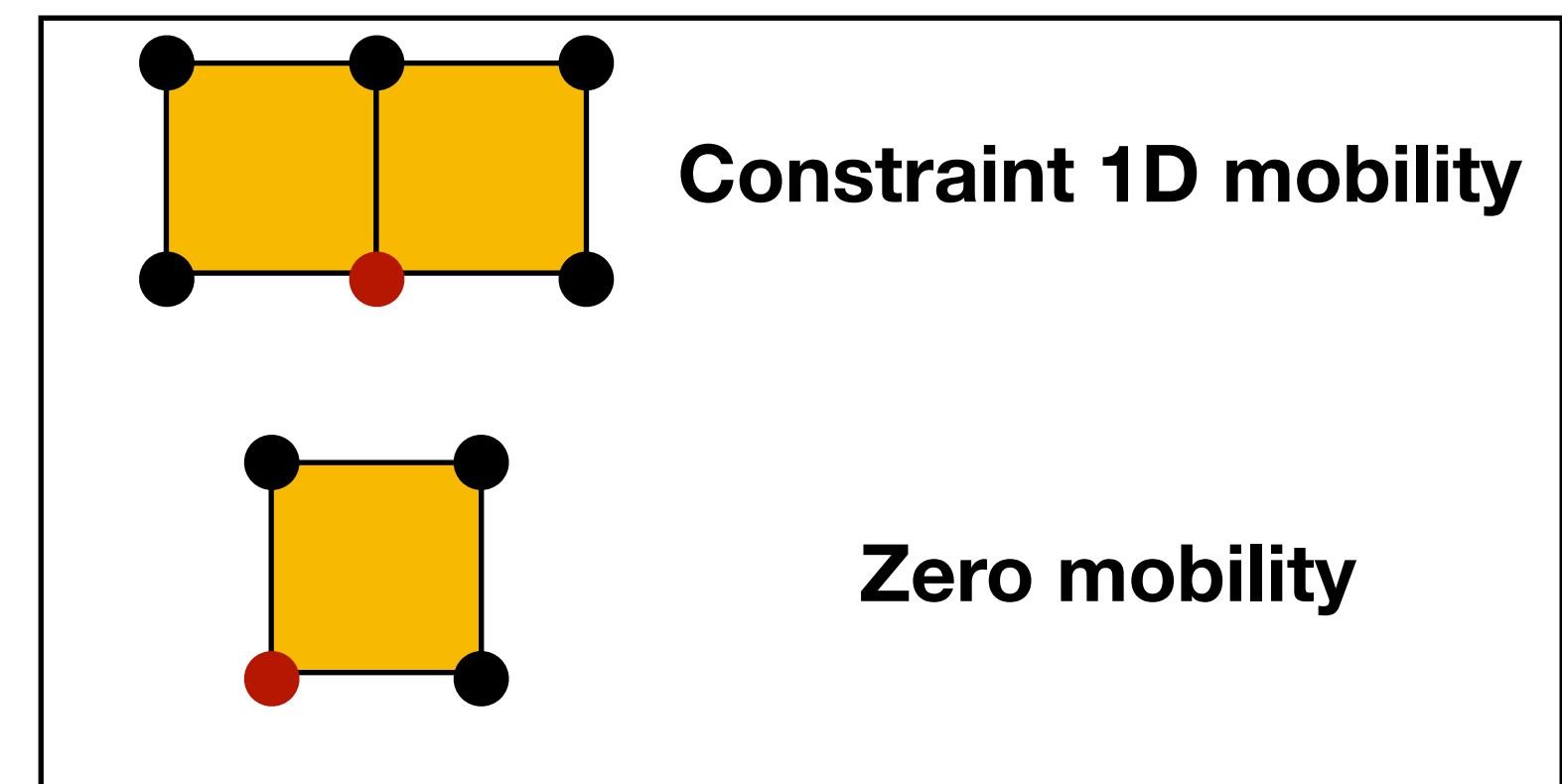
Duality Transformation



$$H_{\text{PIM}} = - \sum_{\square} \tau_i^z \tau_j^z \tau_k^z \tau_l^z - \frac{2h^2}{K_z} \sum_i \tau_i^x$$

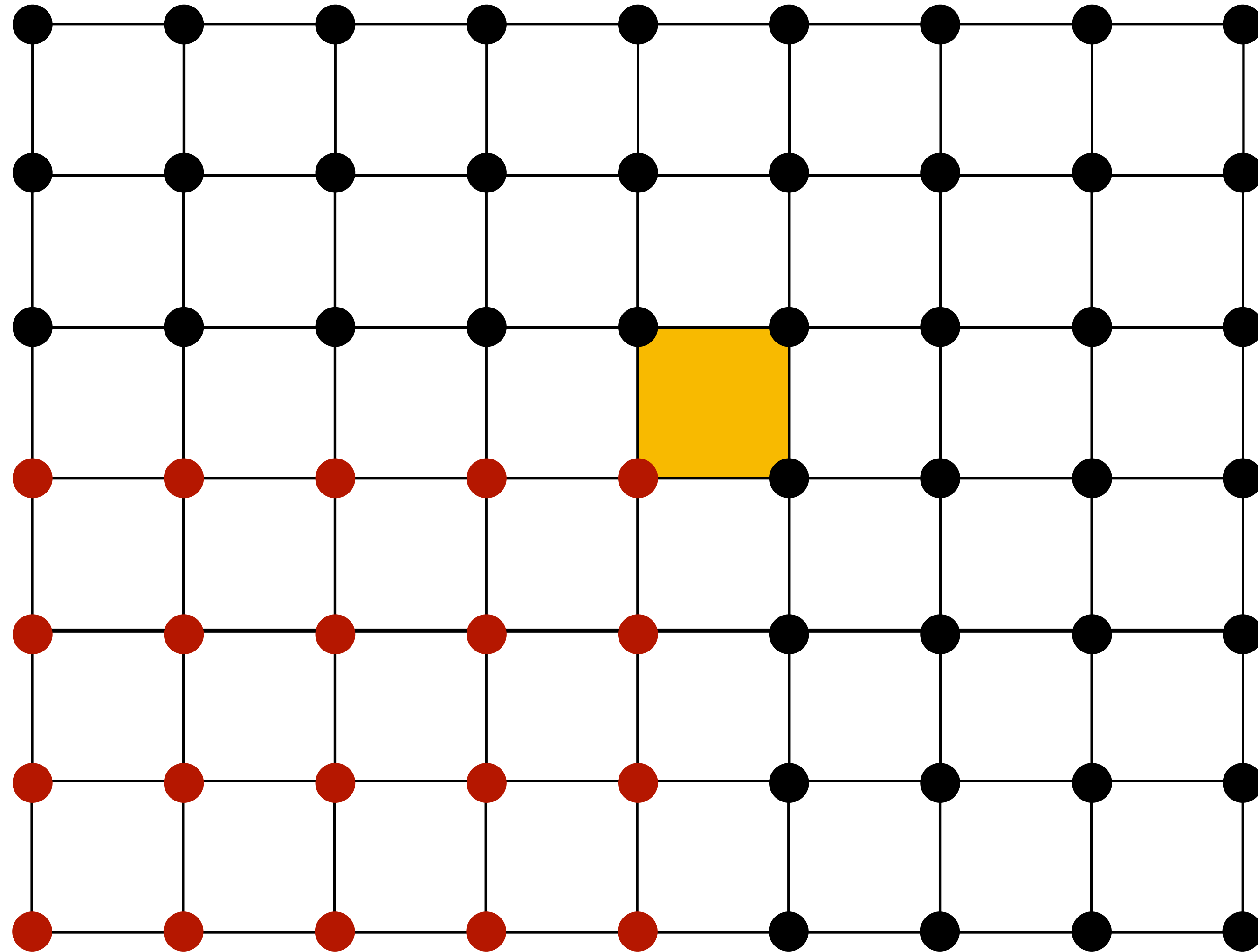


Duality Transformation



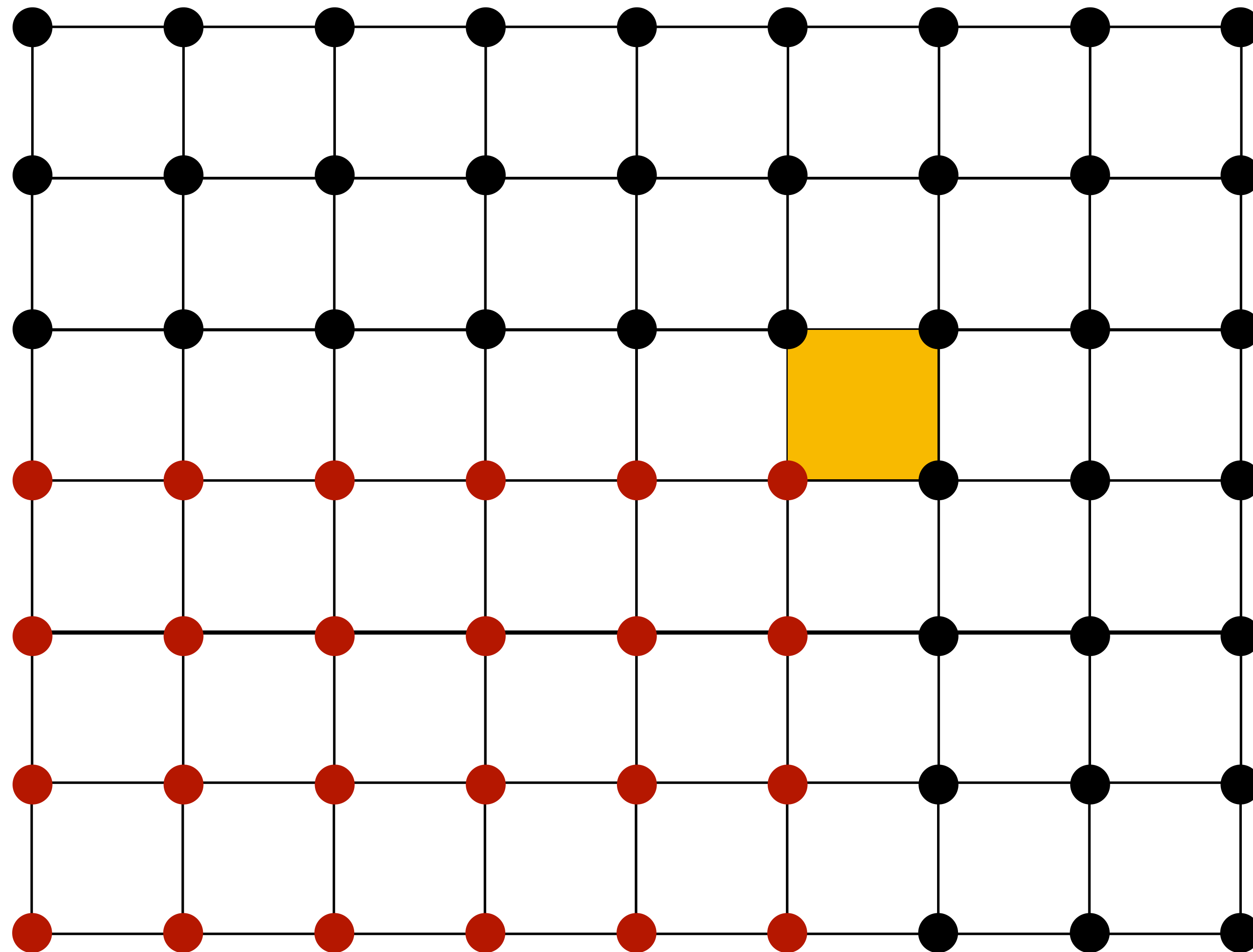
# The Plaquette Ising Model

$$H_{PIM} = - \sum_{\square} \sigma_i^z \sigma_j^z \sigma_k^z \sigma_l^z$$



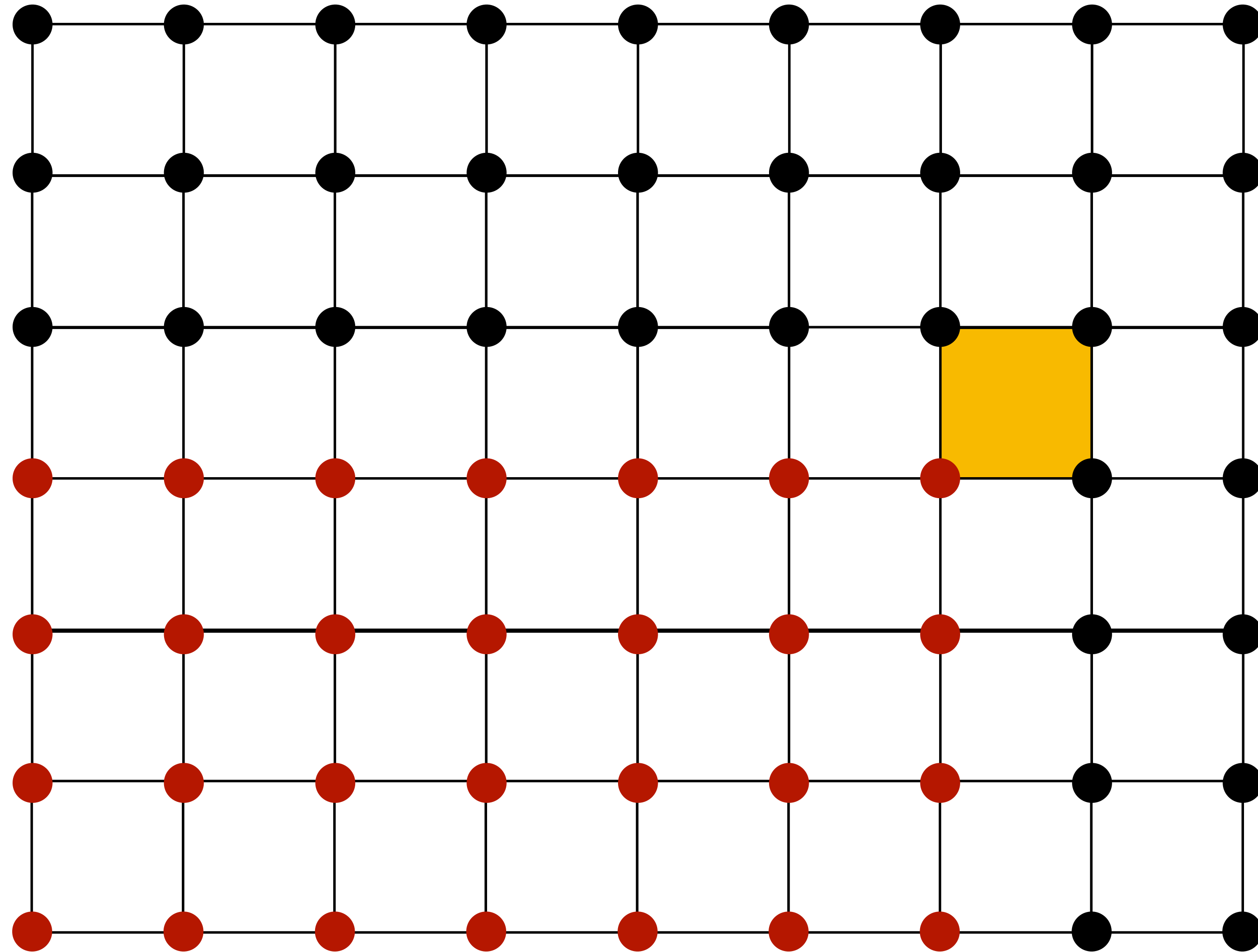
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$$H_{PIM} = - \sum_{\square} \sigma_i^z \sigma_j^z \sigma_k^z \sigma_l^z$$



# The Plaquette Ising Model

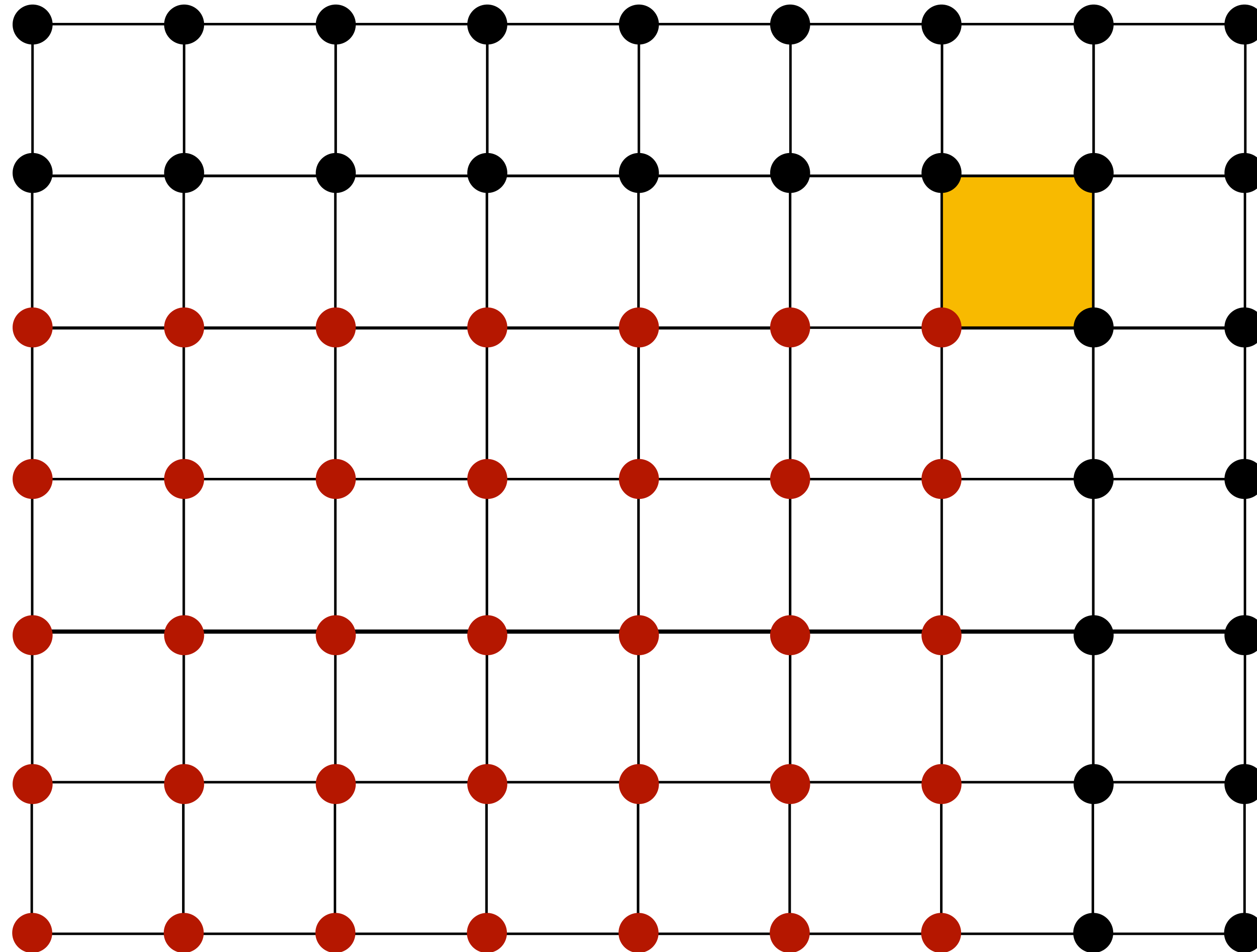
$$H_{PIM} = - \sum_{\square} \sigma_i^z \sigma_j^z \sigma_k^z \sigma_l^z$$





# The Plaquette Ising Model

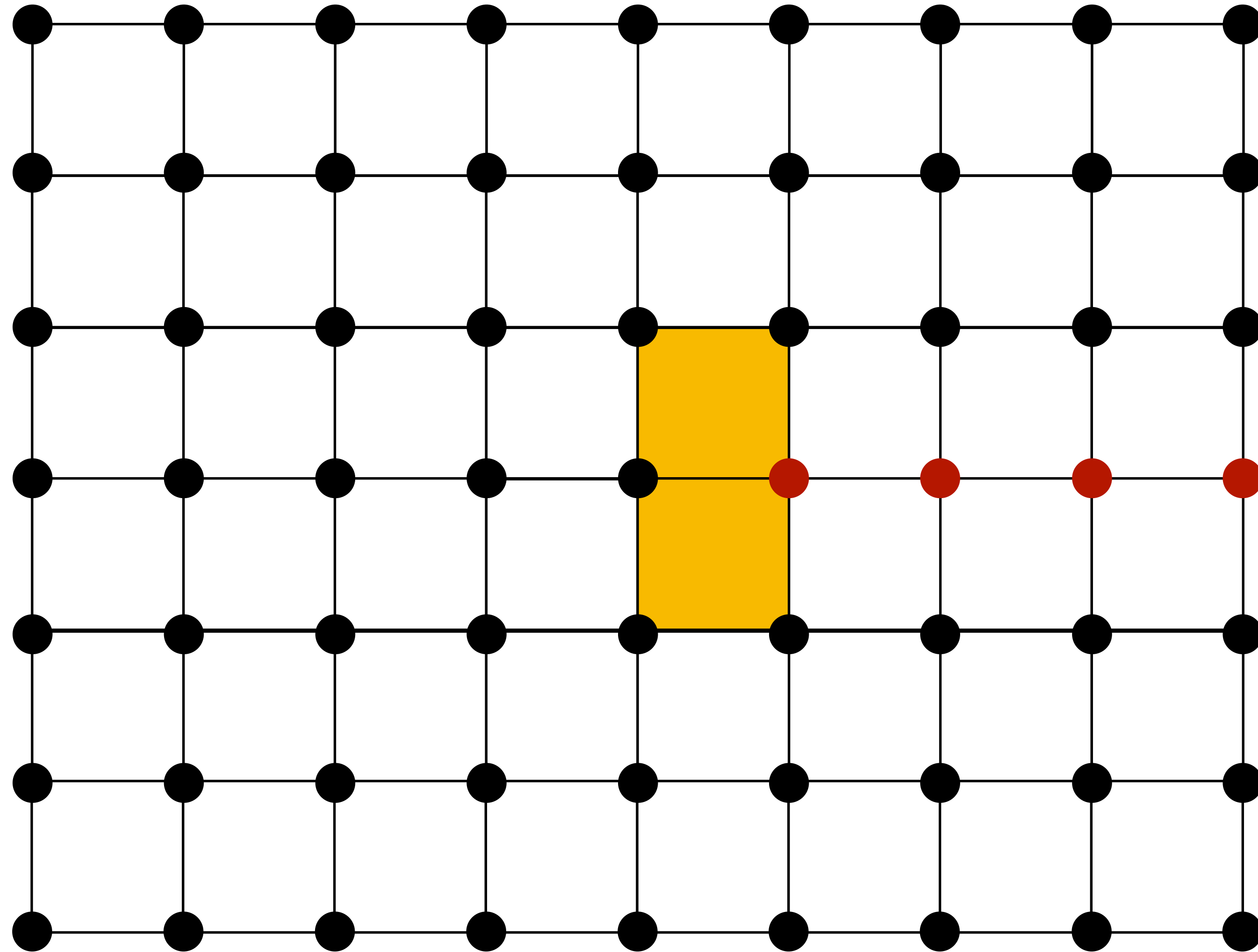
$$H_{PIM} = - \sum_{\square} \sigma_i^z \sigma_j^z \sigma_k^z \sigma_l^z$$



**Infinite  
order  
process**

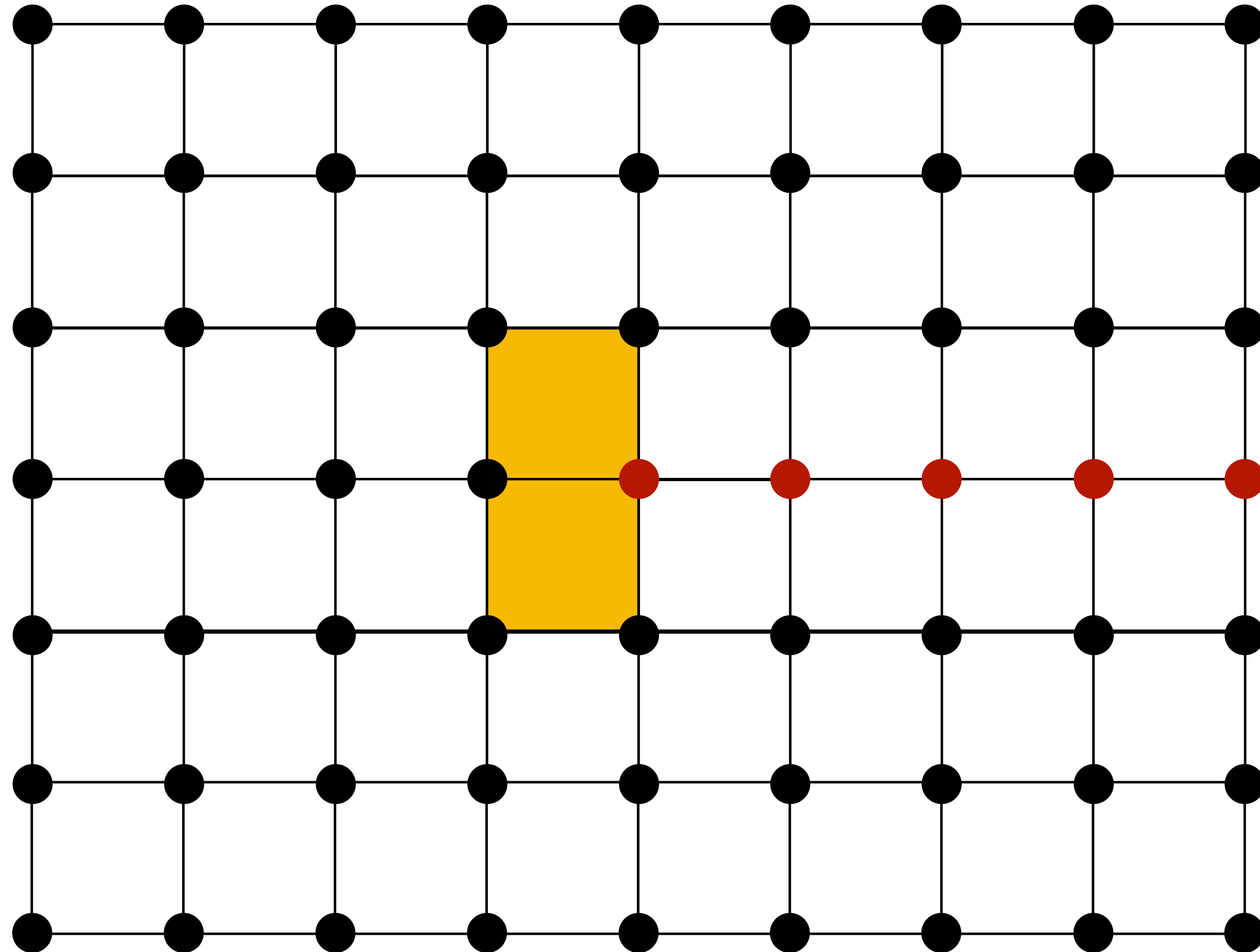
# The Plaquette Ising Model

$$H_{PIM} = - \sum_{\square} \sigma_i^z \sigma_j^z \sigma_k^z \sigma_l^z$$



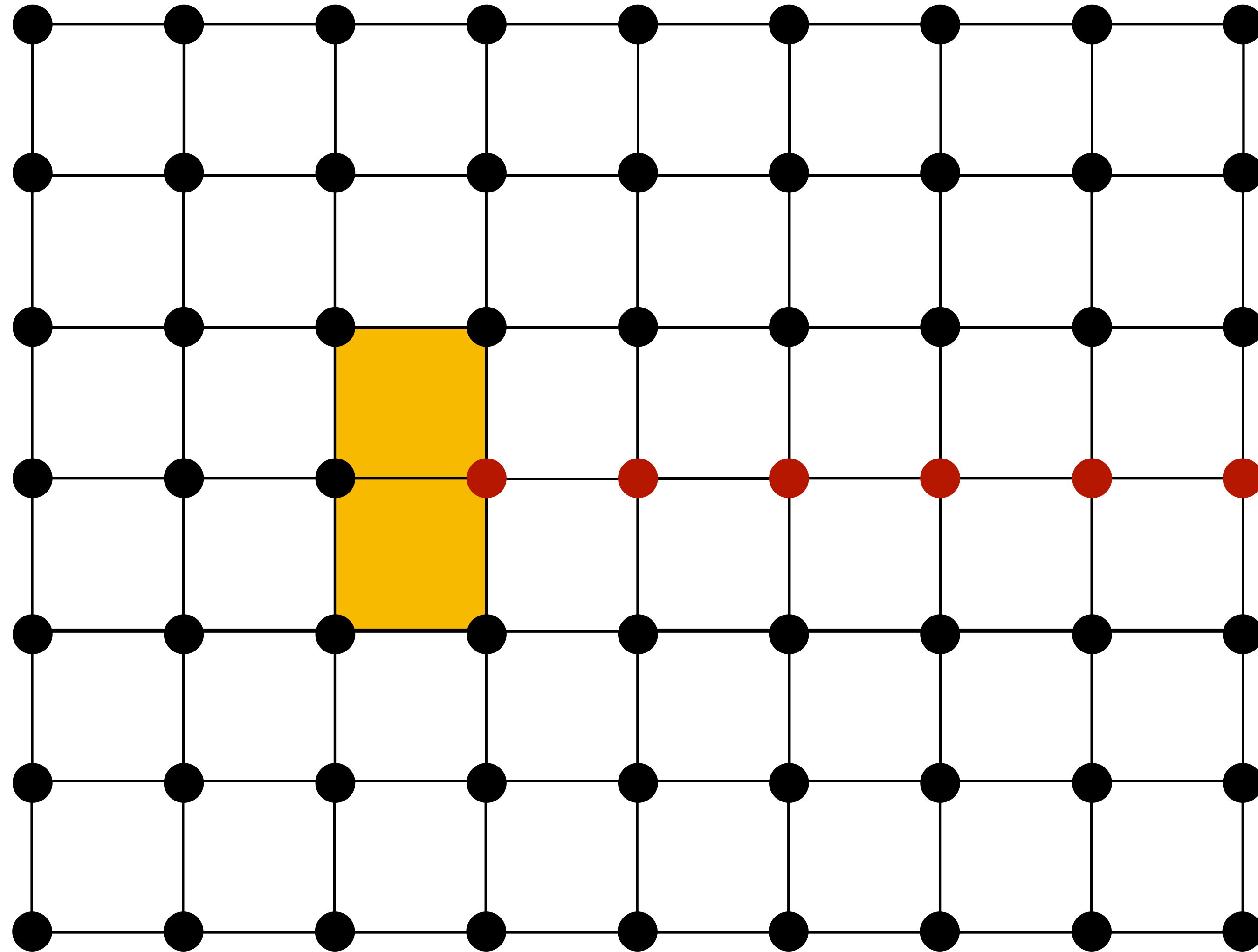
# The Plaquette Ising Model

$$H_{PIM} = - \sum_{\square} \sigma_i^z \sigma_j^z \sigma_k^z \sigma_l^z$$



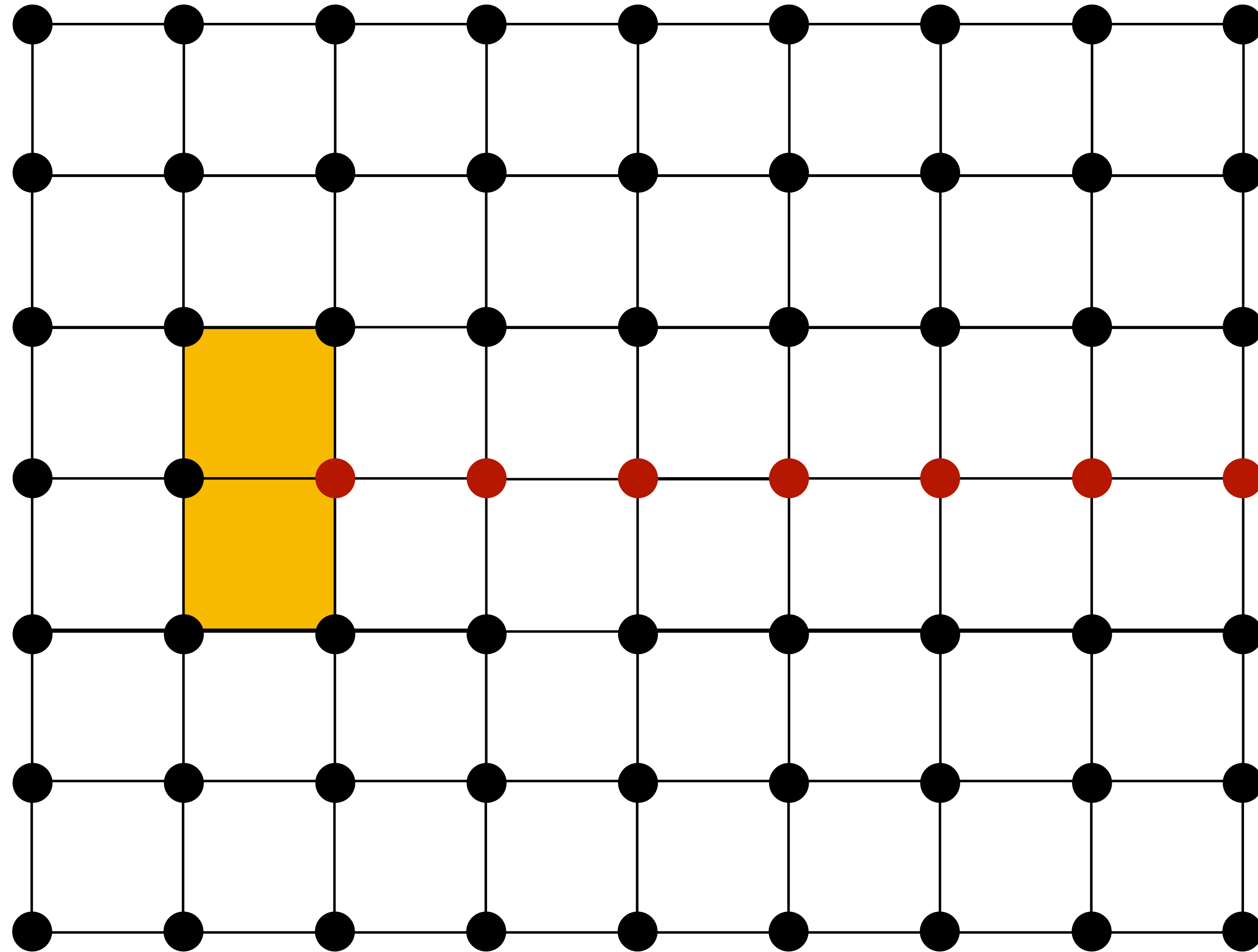
# The Plaquette Ising Model

$$H_{PIM} = - \sum_{\square} \sigma_i^z \sigma_j^z \sigma_k^z \sigma_l^z$$



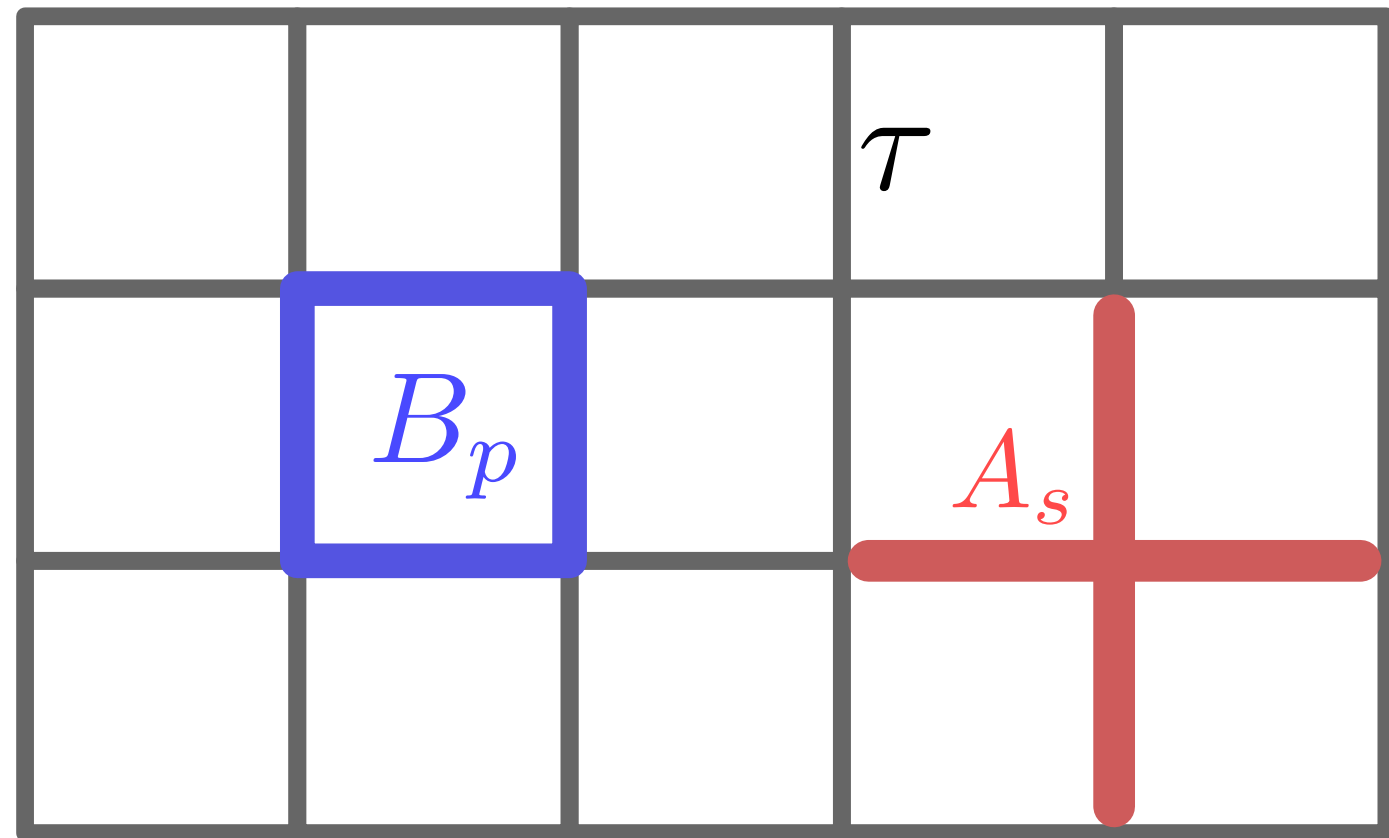
# The Plaquette Ising Model

$$H_{PIM} = - \sum_{\square} \sigma_i^z \sigma_j^z \sigma_k^z \sigma_l^z$$



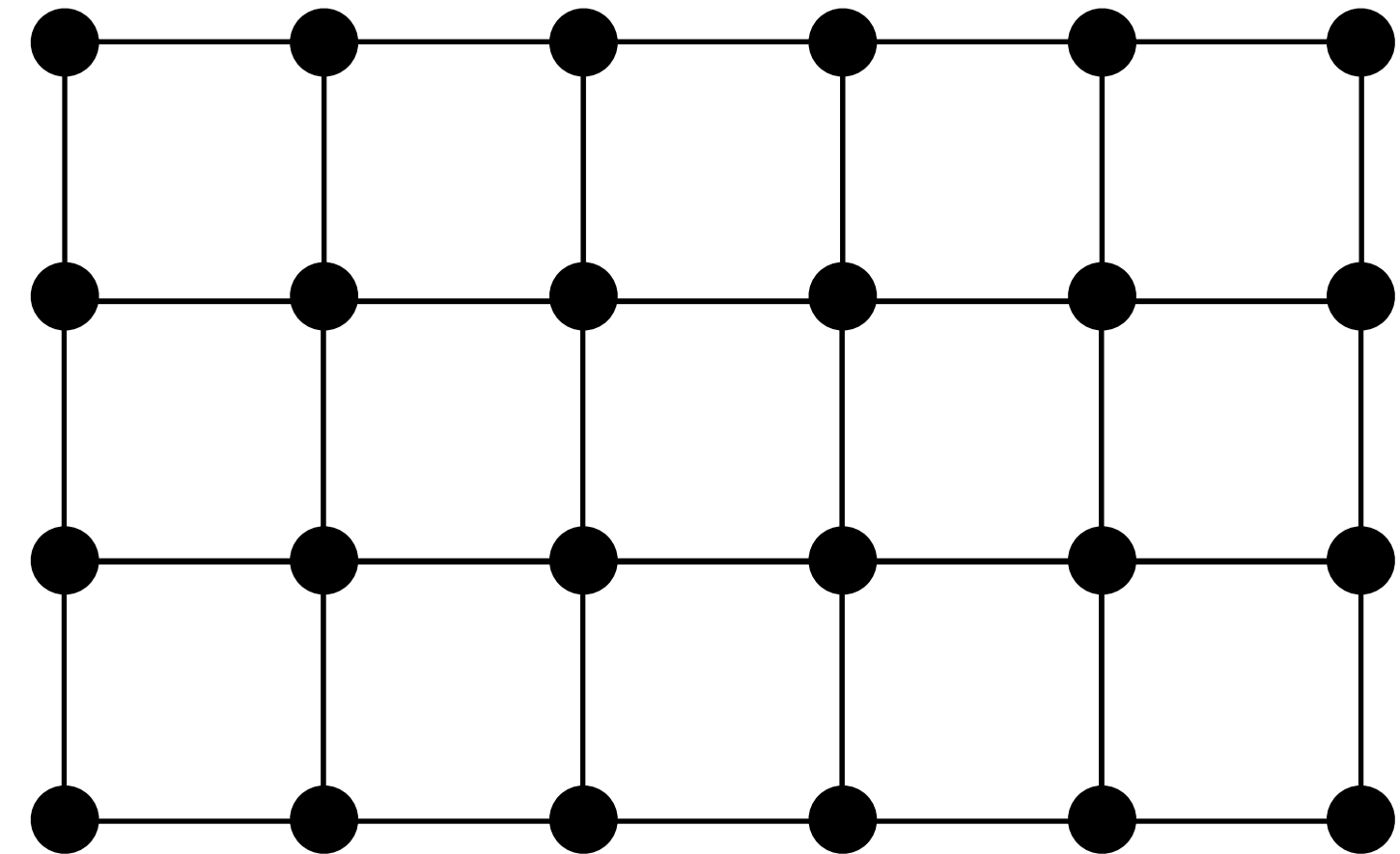
**First order  
process**

# Fractonic Physics by Duality

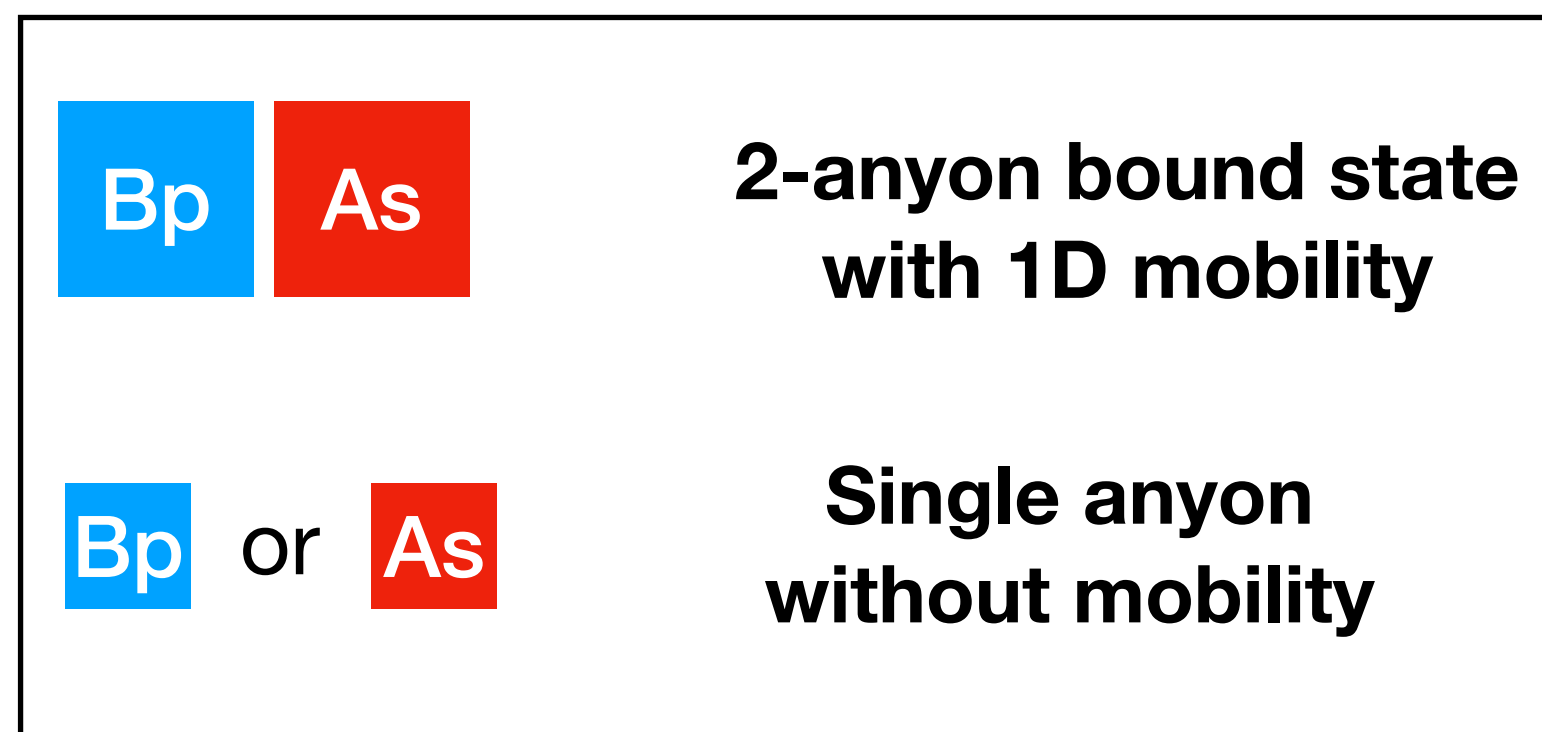


$$H_{K_z \gg K_{x,y}}^{\text{eff}} = H_{\text{TC}} - \frac{2h^2}{K_z} \sum_i \tau_i^y$$

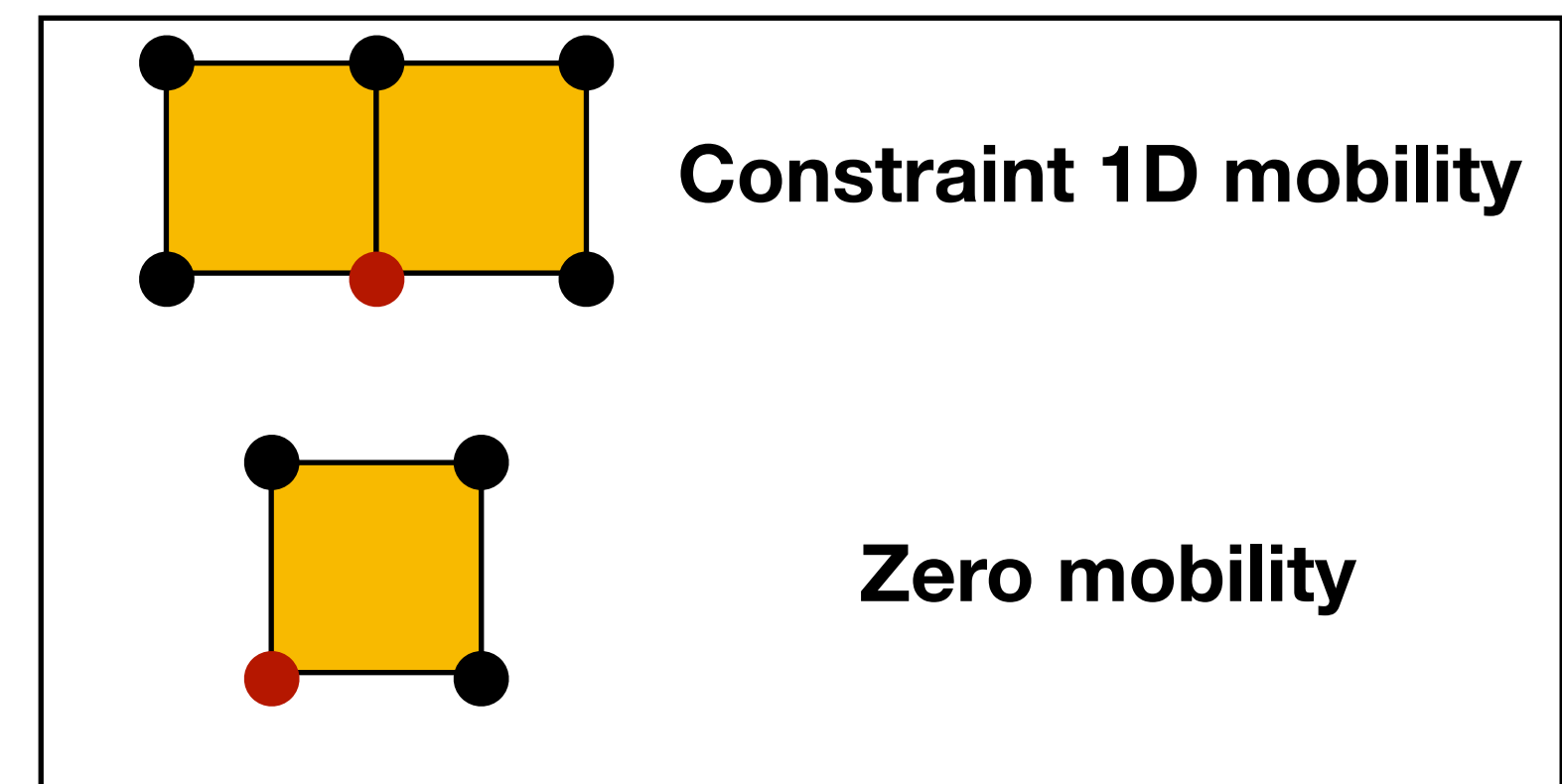
Duality Transformation



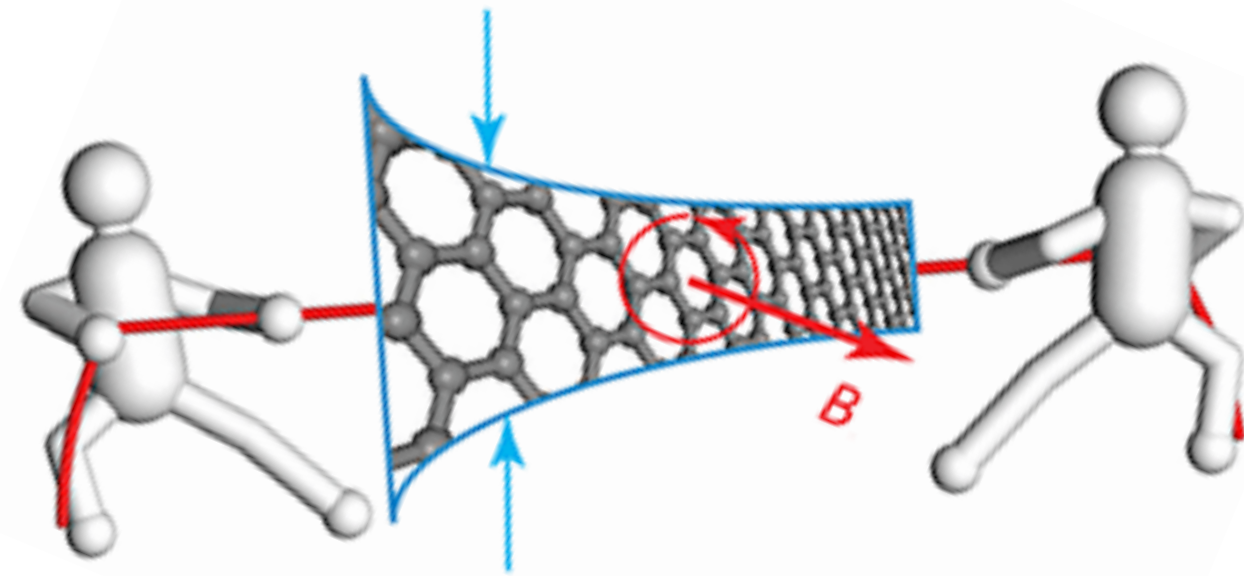
$$H_{\text{PIM}} = - \sum_{\square} \tau_i^z \tau_j^z \tau_k^z \tau_l^z - \frac{2h^2}{K_z} \sum_i \tau_i^x$$



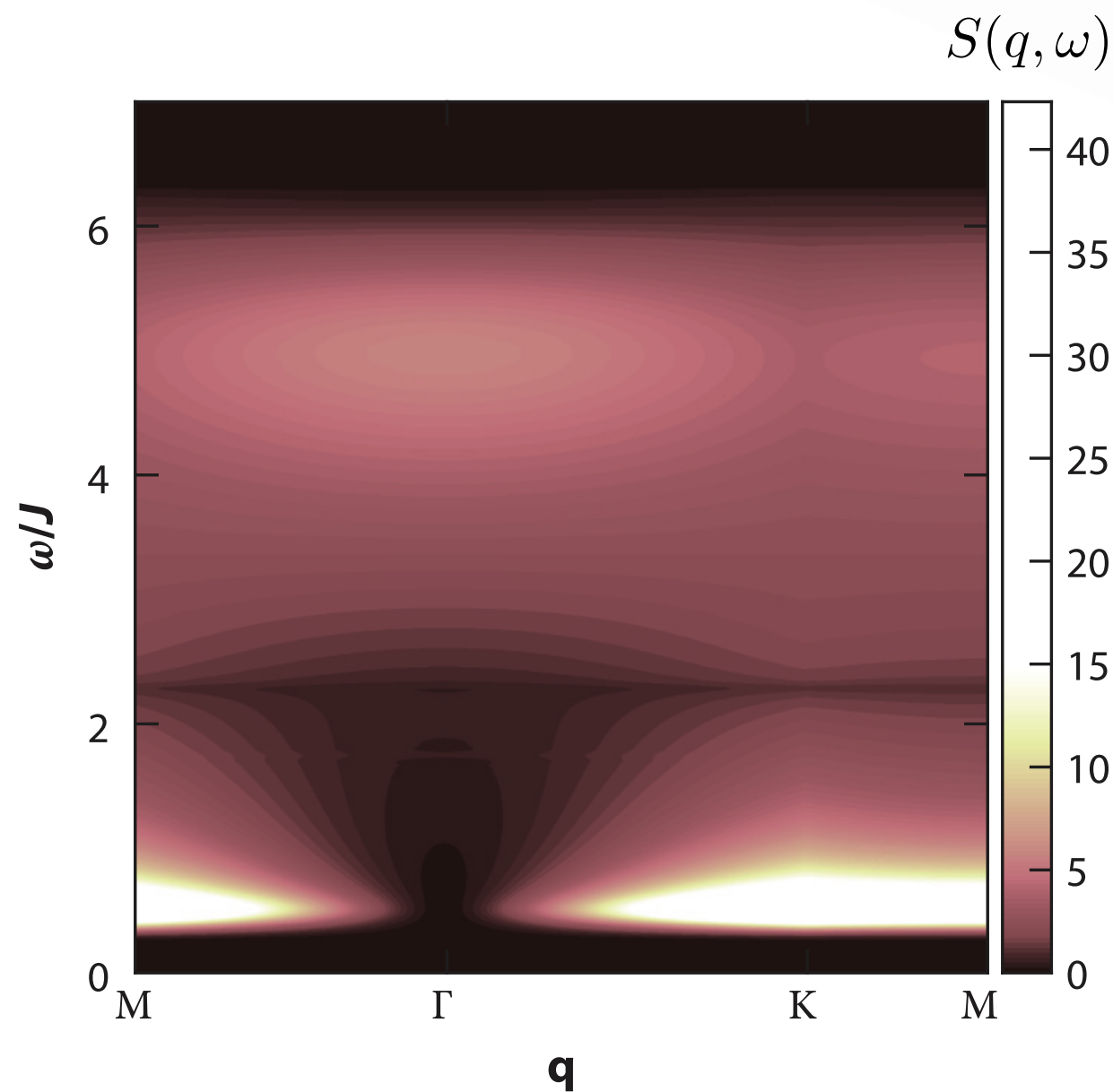
Duality Transformation



# Taking away: Sharp scattering signal of fractionalization

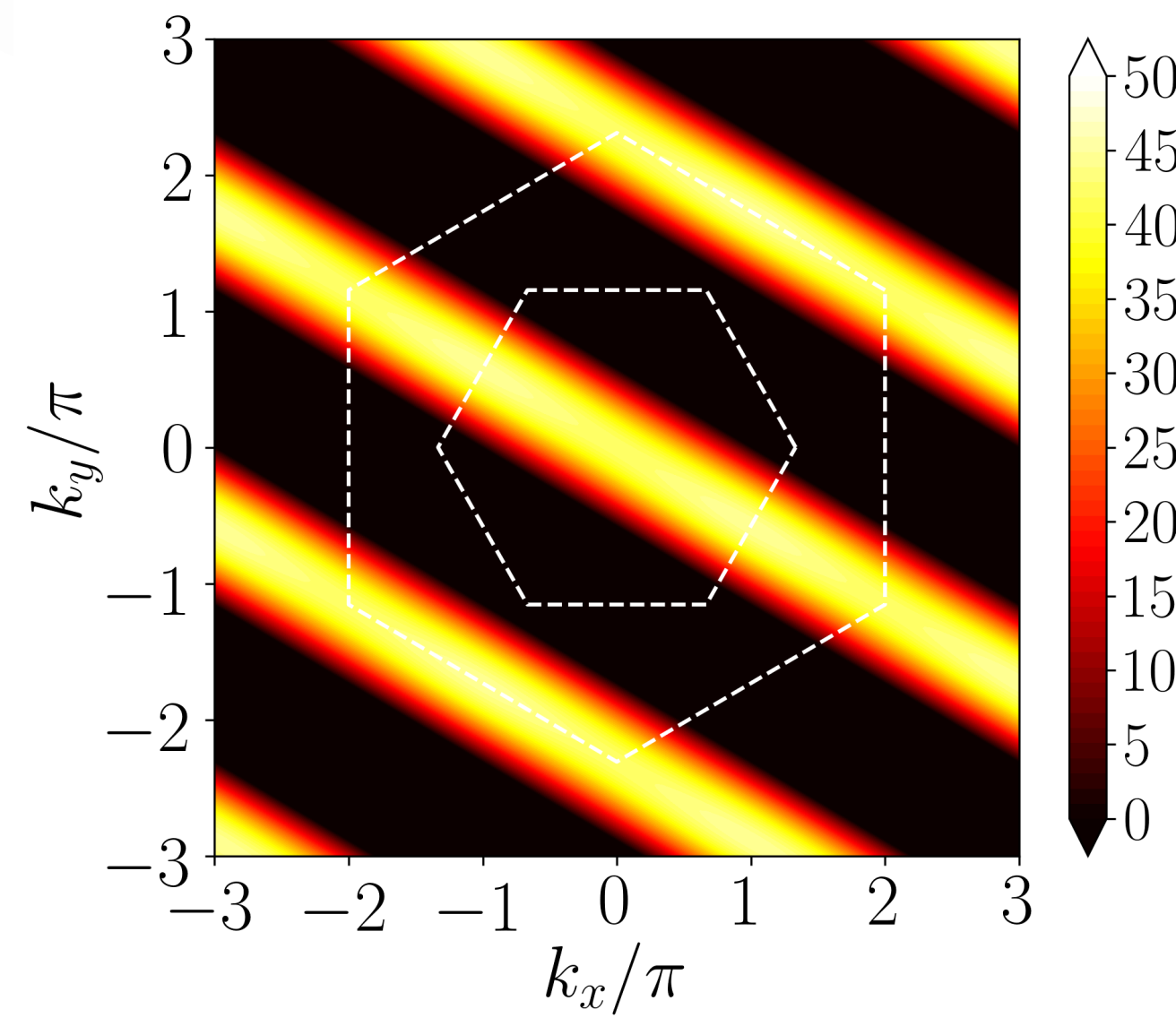


$$H = \sum_x K_x \sigma_i^x \sigma_{i+x}^x + \sum_y K_y \sigma_i^y \sigma_{i+y}^y + \sum_z K_z \sigma_i^z \sigma_{i+z}^z - B \sum_i \sigma_i^{e_3}$$



**1-spin flip**

**fuzzy mix of fractionalized particles**



**2-spin flip**

**Sharp 1D fracton modes**



	<b>Heisenberg</b>	<b>Kitaev QSL (<math>K_z &gt; 2</math>)</b>
<b>1 spin flip</b>	Sharp magnon modes	Continuum of Fractionalized particles
<b>2 spin flip</b>	2-magnon continuum	Sharp Anyon bound state
<b>4 spin flip</b>	4-magnon continuum	Sharp Anyon bound state

**Thanks!**